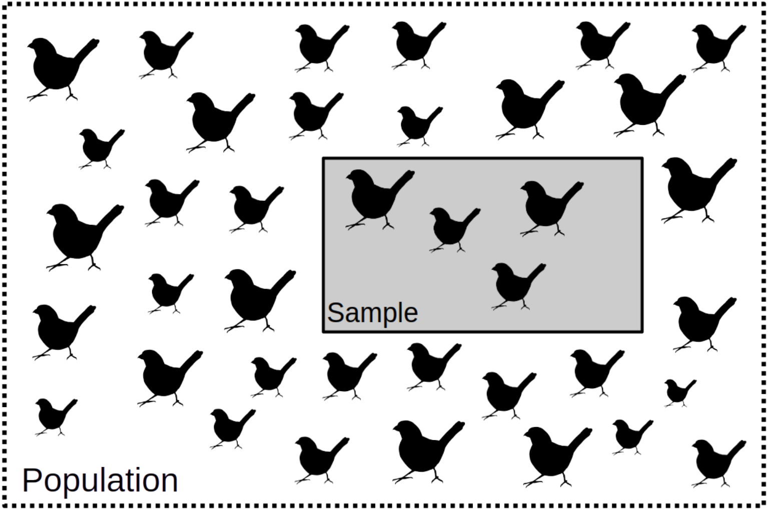
Populations, variables, units, uncertainty

Populations and samples



Types of variables: definitions

- **Categorical:** Fixed number of options
  - Nominal: No inherent order
  - Ordinal: Inherent order
- **Quantitative:** Numbers meaningful
  - Discrete: Limited number of values
  - Continuous: Any real number

All of these measurements have uncertainty

Metrology: The science of measurement

Metrology focuses on measurement accuracy, precision, and units[1]

- **Measurement**: Determination of the properties of a unit of observation

[1] Rabinovich, SG. 2013. Evaluating Measurement Accuracy: A Practical Approach. Springer Science & Business Media. 10.1007/978-3-319-60125-0

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- **Measurement**: Determination of the properties of a unit of observation
- **Measurand**: The unknown *true* value of what we want to measure
- **Measurement error**: Difference between the measurement and the measurand

[1] Rabinovich, SG. 2013. Evaluating Measurement Accuracy: A Practical Approach. Springer Science & Business Media. 10.1007/978-3-319-60125-0

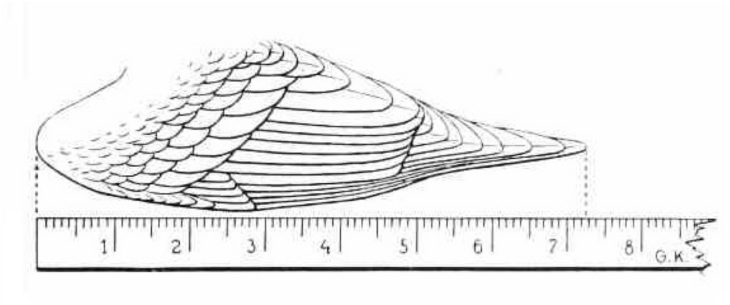
Metrology: The science of measurement

Metrology originally for economic activities[1]

- Need standardised measurements for trade
- Standardised to a measurement **unit:**
  - Unit of length (e.g., cm)
  - Unit of mass (e.g., mg)
  - Unit of time (e.g., seconds)
- The 'unit' defines what is 1

[1] Fanton, JP. 2019. International Journal of Metrology and Quality Engineering. 10:5.

Measuring wing length in centimetres (cm)



- Measure 7.28 *units* of length (cm)
- Measurand is the *true* wing length
- Repeated measures estimate *error*

[1]**Image:** Reichenow, A. 1913. (Public domain).

Accuracy and precision

- **Accuracy** is how close a measurement is to the *true* value we want to measure.[1]

[1] Rabinovich, SG. 2013. Evaluating Measurement Accuracy: A Practical Approach. Springer Science & Business Media. 10.1007/978-3-319-60125-0

[2] Wardlaw, AC. 1985. Practical Statistics for Experimental Biologists (p. 290). John Wiley & Sons, Chichester, UK.

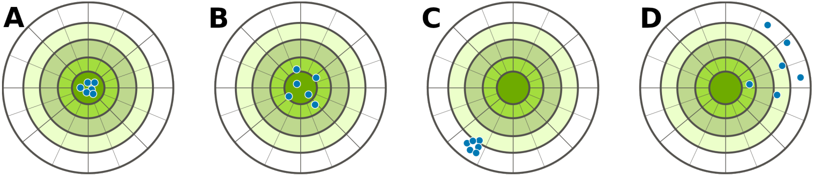
Accuracy and precision

- **Accuracy** is how close a measurement is to the *true* value we want to measure.[1]
- **Precision** is how consistent a measurement will be if replicated multiple times.[2]

[1] Rabinovich, SG. 2013. Evaluating Measurement Accuracy: A Practical Approach. Springer Science & Business Media. 10.1007/978-3-319-60125-0

[2] Wardlaw, AC. 1985. Practical Statistics for Experimental Biologists (p. 290). John Wiley & Sons, Chichester, UK.

Accuracy and precision



Measurements can be as follows:

- **A:** Accurate and precise
- **B:** Accurate but not precise
- **C:** Not accurate but precise
- **D:** Not accurate nor precise

[1] **Image:** Willighagen, E. 2014. (Public domain).

Units of measurement

Units of *measurement* different from units of *observation*

- Measurement unit is a defined measurement quantity
- Standardised units of measurement ensure accuracy[1]
- Ideally based on fundamental constants of nature[2]
- Measurement units can be base or derived
- Seven standardised base units

[1] Quinn, TJ. 1995. Metrologia, 31:515–527. 10.1088/0026-1394/31/6/011
[2] Stock, M, et al. 2019. Metrologia, 56:022001

Kilogram was *defined* by a mass of metal

[1]**Image:** National Inst. of Standards & Technology. 2021. (Public domain).

Kilogram was *defined* by a mass of metal

Standardising mass to a physical object problematic

- If object changes, so does the unit of mass
- Base unit affects all other measurements

[1] Stock, M et al. 2019. Metrologia 56:022001.

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Kilogram redefined in 2019[1]:

- Atomic transition frequency ($\Delta\nu_{Cs}$)
- Speed of light (c)
- Planck constant (h)

$$1\ kg = \left(1.4755213 \times 10^{40}\right) \frac{h\Delta\nu_{Cs}}{c^2}$$

[1] Stock, M et al. 2019. Metrologia 56:022001.

Base units of SI measurements

Measured Quantity	Name of SI Unit	Symbol
Mass	kilogram	kg
Length	metre	m
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Amount of a substance	mole	mol
Luminous intensity	candela	cd

Derived SI measurements

Measured Quantity	Name of Unit	Symbol	Definition in SI Units
Area	square metre	A	m^2
Volume	cubic metre	V	m^3
Speed	metre per second	v	$m\ s^{-1}$
Force	newton	N	$m\ kg\ s^{-2}$
Pressure	pascal	Pa	$m^{-1}\ kg\ s^{-2}$
Energy	joule	J	$m^2\ kg\ s^{-2}$

Derived units of measurement built from base units

Units versus labels

- **Unit** defines a magnitude of a quantity
  - 1 kilogram
  - 1 metre
- **Label** describes the data type
  - soil mass
  - flight distance
- **Counts** do not have units
  - 20 glaciers
  - 800 seeds

Measurement uncertainty propagation

Nothing measured with perfect accuracy

- Noise in the measuring environment
- Mistakes made in measurement
- Limitations of measuring device

Measurement errors accumulate!

[1]**Image:** Perkins, D. 2015. (Public domain).

Measurement uncertainty propogation

Suppose we measured 2 stones separately

Each measurement has a ± error

[1]**Image:** Nijaki, N. 2011. (Public domain).

Combined measurement error

- Stone 1: $40 \pm 1.2\ kg$
- Stone 2: $36 \pm 1.1\ kg$

Combined mass:

$40\ kg + 36\ kg = 76\ kg$

Combined measurement error?

Measurement uncertainty propogation

Stone example:

$$Combined\ mass = Mass\ 1 + Mass\ 2$$

Measurement uncertainty propogation

Stone example:

$$Combined\ mass = Mass\ 1 + Mass\ 2$$

More generally:

$$Z = X + Y$$

Measurement uncertainty propogation

Stone example:

$$\textit{Combined mass} = \textit{Mass}\ 1 + \textit{Mass}\ 2$$

More generally:

$$Z = X + Y$$

With error (E):

$$(Z \pm E_Z) = (X \pm E_X) + (Y \pm E_Y)$$

Measurement uncertainty propogation

$$(Z \pm E_Z) = (X \pm E_X) + (Y \pm E_Y)$$

If we solve for E_Z,

$$E_Z = \sqrt{E_X^2 + E_Y^2}.$$

For our two stones,

$$E_Z = \sqrt{1.2^2 + 1.1^2} = 1.63.$$

Measurement uncertainty propogation

- Morning run: $4.5 \pm 0.3\ km$
- Evening run: $3.8 \pm 0.2\ km$

Combined error (E_Z) for total run length?

$$E_Z = \sqrt{E_X^2 + E_Y^2}.$$

Measurement uncertainty propogation

$$Z = X \times Y$$

Combining errors different for multiplication

$$Z \pm E_Z = (X \pm E_X)(Y \pm E_Y).$$

Measurement uncertainty propogation

$$Z = X \times Y$$

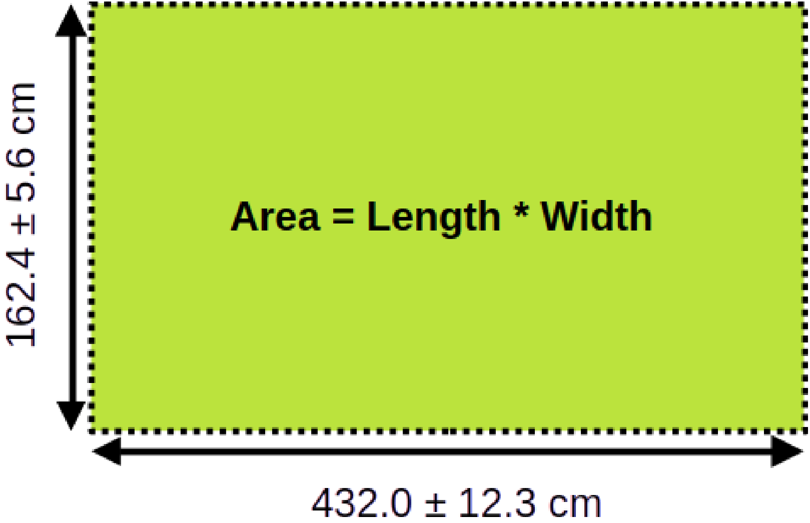
Combining errors different for multiplication

$$Z \pm E_Z = (X \pm E_X)(Y \pm E_Y).$$

If we isolate E_Z,

$$E_Z = Z\sqrt{\left(\frac{E_X}{X}\right)^2 + \left(\frac{E_Y}{Y}\right)^2}.$$

Measurement uncertainty propogation



Calculating uncertainty

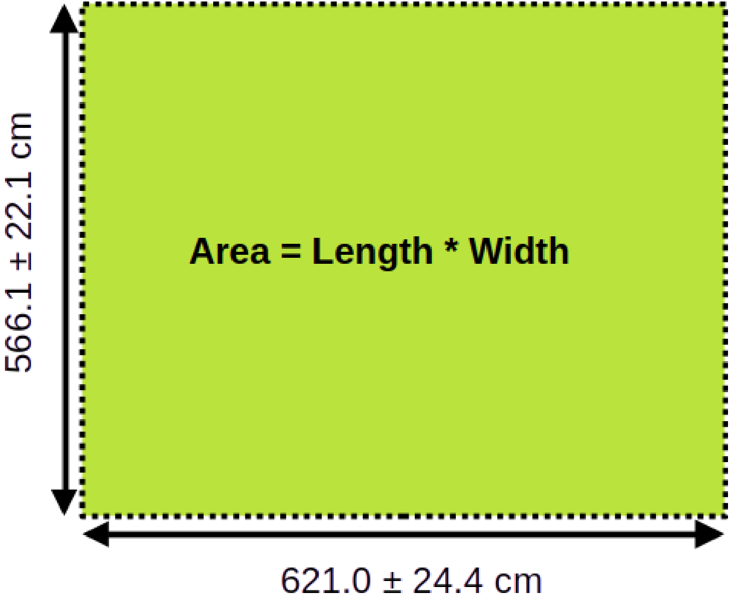
$$E_Z = Z\sqrt{\left(\frac{E_X}{X}\right)^2 + \left(\frac{E_Y}{Y}\right)^2}.$$

- $X = 432$
- $Y = 162$
- $E_X = 12.3$
- $E_Y = 5.6$

Calculating uncertainty

$$E_Z = Z\sqrt{\left(\frac{E_X}{X}\right)^2 + \left(\frac{E_Y}{Y}\right)^2}.$$

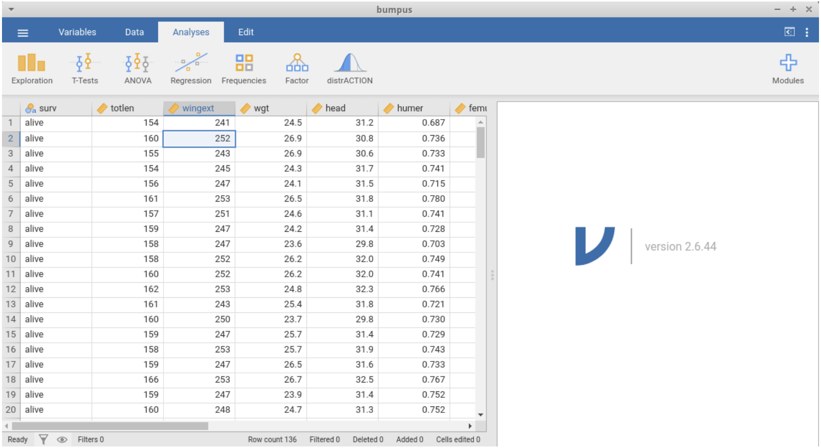
Measurement uncertainty propogation



Calculating uncertainty

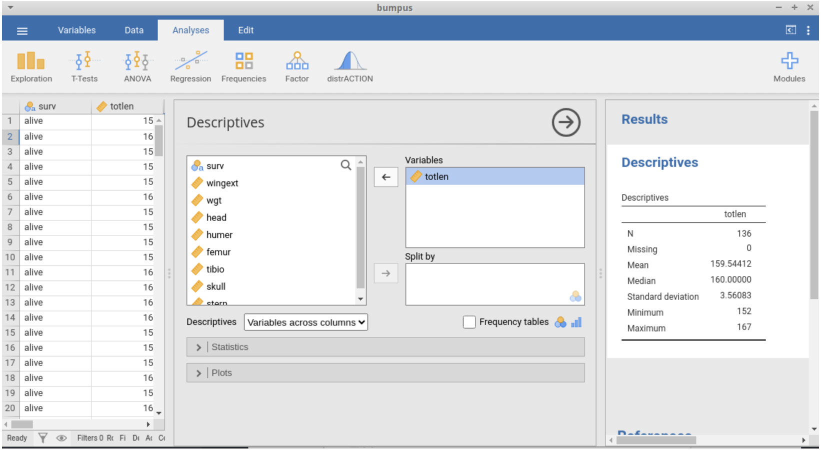
$$E_Z = Z\sqrt{\left(\frac{E_X}{X}\right)^2 + \left(\frac{E_Y}{Y}\right)^2}.$$

Working with jamovi



	surv	totlen	wingext	wgt	head	humer	femu
1	alive	154	241	24.5	31.2	0.687	
2	alive	160	252	26.9	30.8	0.736	
3	alive	155	243	26.9	30.6	0.733	
4	alive	154	245	24.3	31.7	0.741	
5	alive	156	247	24.1	31.5	0.715	
6	alive	161	253	26.5	31.8	0.780	
7	alive	157	251	24.6	31.1	0.741	
8	alive	159	247	24.2	31.4	0.728	
9	alive	158	247	23.6	29.8	0.703	
10	alive	158	252	26.2	32.0	0.749	
11	alive	160	252	26.2	32.0	0.741	
12	alive	162	253	24.8	32.3	0.766	
13	alive	161	243	25.4	31.8	0.721	
14	alive	160	250	23.7	29.8	0.730	
15	alive	159	247	25.7	31.4	0.729	
16	alive	158	253	25.7	31.9	0.743	
17	alive	159	247	26.5	31.6	0.733	
18	alive	166	253	26.7	32.5	0.767	
19	alive	159	247	23.9	31.4	0.752	
20	alive	160	248	24.7	31.3	0.752	

Working with jamovi



Working with jamovi