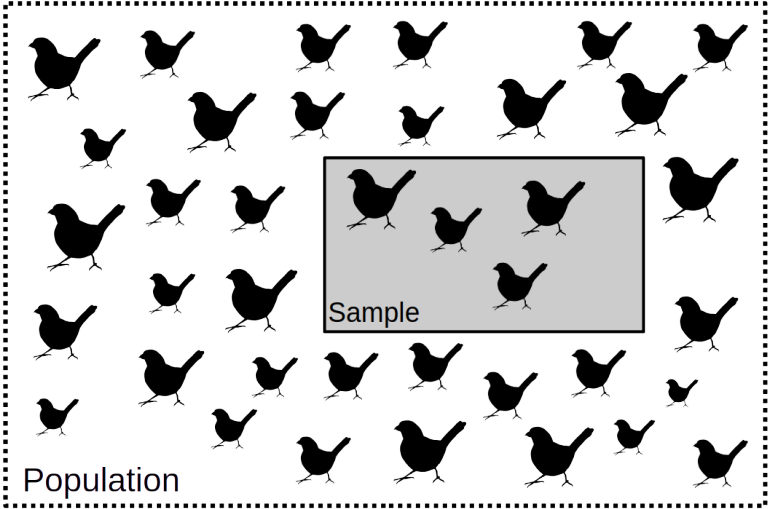


Populations, variables, units, uncertainty

Populations and samples



Types of variables: definitions

- ▶ **Categorical:** Fixed number of options
 - ▶ Nominal: No inherent order
 - ▶ Ordinal: Inherent order
- ▶ **Quantitative:** Numbers meaningful
 - ▶ Discrete: Limited number of values
 - ▶ Continuous: Any real number

All of these measurements have uncertainty

Metrology: The science of measurement

Metrology focuses on measurement accuracy, precision, and units¹

- ▶ **Measurement:** Determination of the properties of a unit of observation

¹Rabinovich, SG. 2013. Evaluating Measurement Accuracy: A Practical Approach. Springer Science & Business Media. [10.1007/978-3-319-60125-0](https://doi.org/10.1007/978-3-319-60125-0)

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- ▶ **Measurement:** Determination of the properties of a unit of observation
- ▶ **Measurand:** The unknown *true* value of what we want to measure
- ▶ **Measurement error:** Difference between the measurement and the measurand

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Metrology: The science of measurement

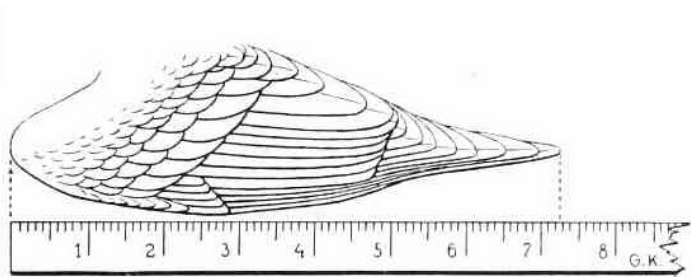
Metrology originally for economic activities¹

- ▶ Need standardised measurements for trade
- ▶ Standardised to a measurement **unit**:
 - ▶ Unit of length (e.g., cm)
 - ▶ Unit of mass (e.g., mg)
 - ▶ Unit of time (e.g., seconds)
- ▶ The 'unit' defines what is 1

¹Fanton, JP. 2019. International Journal of Metrology and Quality Engineering. [10:5](#).

Metrology: The science of measurement

Measuring wing length in centimetres (cm)



- ▶ Measure 7.28 *units* of length (cm)
- ▶ Measurand is the *true* wing length
- ▶ Repeated measures estimate *error*

¹Image: Reichenow, A. 1913. (Public domain).

- ▶ **Accuracy** is how close a measurement is to the *true* value we want to measure.¹

¹Rabinovich, SG. 2013. Evaluating Measurement Accuracy: A Practical Approach. Springer Science & Business Media. [10.1007/978-3-319-60125-0](https://doi.org/10.1007/978-3-319-60125-0)

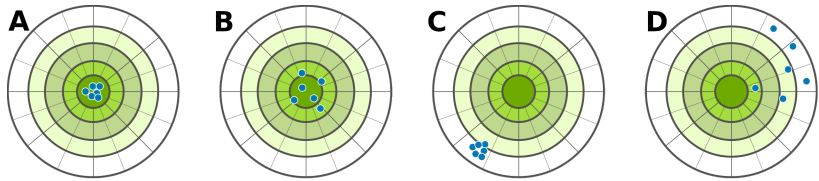
²Wardlaw, AC. 1985. Practical Statistics for Experimental Biologists (p. 290). John Wiley & Sons, Chichester, UK.

- ▶ **Accuracy** is how close a measurement is to the *true* value we want to measure.¹
- ▶ **Precision** is how consistent a measurement will be if replicated multiple times.²

¹Rabinovich, SG. 2013. Evaluating Measurement Accuracy: A Practical Approach. Springer Science & Business Media. [10.1007/978-3-319-60125-0](https://doi.org/10.1007/978-3-319-60125-0)

²Wardlaw, AC. 1985. Practical Statistics for Experimental Biologists (p. 290). John Wiley & Sons, Chichester, UK.

Accuracy and precision



Measurements can be as follows:

- ▶ **A:** Accurate and precise
- ▶ **B:** Accurate but not precise
- ▶ **C:** Not accurate but precise
- ▶ **D:** Not accurate nor precise

Units of measurement

Units of *measurement* different from units of *observation*

- ▶ Measurement unit is a defined measurement quantity
- ▶ Standardised units of measurement ensure accuracy¹
- ▶ Ideally based on fundamental constants of nature²
- ▶ Measurement units can be base or derived
- ▶ Seven standardised base units

¹Quinn, T.J. 1995. Metrologia, 31:515–527. [10.1088/0026-1394/31/6/011](https://doi.org/10.1088/0026-1394/31/6/011)

²Stock, M, et al. 2019. Metrologia, 56:022001

Kilogram was *defined* by a mass of metal



¹Image: National Inst. of Standards & Technology. 2021. (Public domain).

Kilogram was *defined* by a mass of metal

Standardising mass to a physical object problematic

- ▶ If object changes, so does the unit of mass
- ▶ Base unit affects all other measurements

¹Stock, M et al. 2019. Metrologia [56:022001](#).

Kilogram was *defined* by a mass of metal

Standardising mass to a physical object problematic

- ▶ If object changes, so does the unit of mass
- ▶ Base unit affects all other measurements

Kilogram redefined in 2019¹:

- ▶ Atomic transition frequency ($\Delta\nu_{Cs}$)
- ▶ Speed of light (c)
- ▶ Planck constant (h)

$$1 \text{ kg} = (1.4755213 \times 10^{40}) \frac{h\Delta\nu_{Cs}}{c^2}$$

¹Stock, M et al. 2019. Metrologia [56:022001](#).

Base units of SI measurements

Measured Quantity	Name of SI Unit	Symbol
Mass	kilogram	kg
Length	metre	m
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Amount of a substance	mole	mol
Luminous intensity	candela	cd

Derived SI measurements

Measured Quantity	Name of Unit	Symbol	Definition in SI Units
Area	square metre	<i>A</i>	m^2
Volume	cubic metre	<i>V</i>	m^3
Speed	metre per second	<i>v</i>	m s^{-1}
Force	newton	<i>N</i>	m kg s^{-2}
Pressure	pascal	<i>Pa</i>	$\text{m}^{-1} \text{kg s}^{-2}$
Energy	joule	<i>J</i>	$\text{m}^2 \text{kg s}^{-2}$

Derived units of measurement built from base units

Units versus labels

- ▶ **Unit** defines a magnitude of a quantity
 - ▶ 1 kilogram
 - ▶ 1 metre
- ▶ **Label** describes the data type
 - ▶ soil mass
 - ▶ flight distance
- ▶ **Counts** do not have units
 - ▶ 20 glaciers
 - ▶ 800 seeds

Measurement uncertainty propagation

Nothing measured with perfect accuracy

- ▶ Noise in the measuring environment
- ▶ Mistakes made in measurement
- ▶ Limitations of measuring device

Measurement errors accumulate!



¹Image: Perkins, D. 2015. ([Public domain](#)).

Measurement uncertainty propagation

Suppose we measured 2 stones separately



Each measurement has a \pm error

¹Image: Nijaki, N. 2011. ([Public domain](#)).

Combined measurement error

- ▶ Stone 1: $40 \pm 1.2 \text{ kg}$
- ▶ Stone 2: $36 \pm 1.1 \text{ kg}$

Combined mass:

$$40 \text{ kg} + 36 \text{ kg} = 76 \text{ kg}$$

Combined measurement error?

Measurement uncertainty propagation

Stone example:

$$\textit{Combined mass} = \textit{Mass 1} + \textit{Mass 2}$$

Measurement uncertainty propagation

Stone example:

$$\textit{Combined mass} = \textit{Mass 1} + \textit{Mass 2}$$

More generally:

$$Z = X + Y$$

Measurement uncertainty propagation

Stone example:

$$\textit{Combined mass} = \textit{Mass 1} + \textit{Mass 2}$$

More generally:

$$Z = X + Y$$

With error (E):

$$(Z \pm E_Z) = (X \pm E_X) + (Y \pm E_Y)$$

Measurement uncertainty propagation

$$(Z \pm E_Z) = (X \pm E_X) + (Y \pm E_Y)$$

If we solve for E_Z ,

$$E_Z = \sqrt{E_X^2 + E_Y^2}.$$

For our two stones,

$$E_Z = \sqrt{1.2^2 + 1.1^2} = 1.63.$$

Measurement uncertainty propagation

- ▶ Morning run: $4.5 \pm 0.3 \text{ km}$
- ▶ Evening run: $3.8 \pm 0.2 \text{ km}$

Combined error (E_Z) for total run length?

$$E_Z = \sqrt{E_X^2 + E_Y^2}.$$

Measurement uncertainty propagation

$$Z = X \times Y$$

Combining errors different for multiplication

$$Z \pm E_Z = (X \pm E_X)(Y \pm E_Y).$$

Measurement uncertainty propagation

$$Z = X \times Y$$

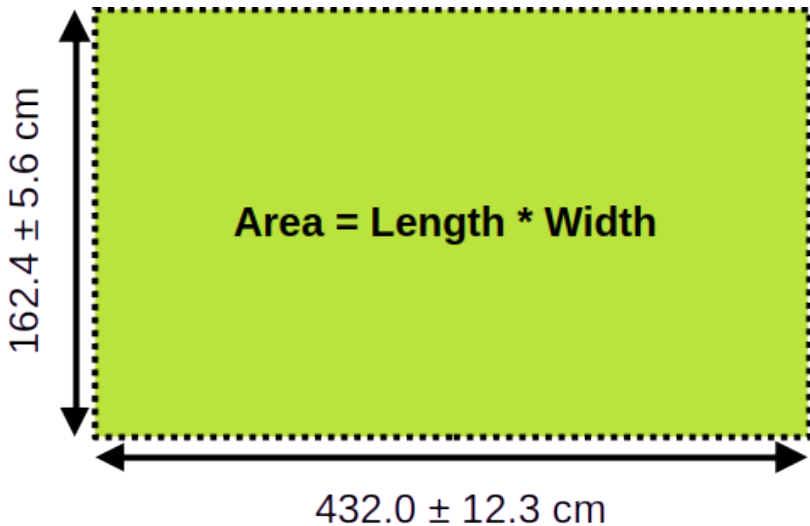
Combining errors different for multiplication

$$Z \pm E_Z = (X \pm E_X)(Y \pm E_Y).$$

If we isolate E_Z ,

$$E_Z = Z \sqrt{\left(\frac{E_X}{X}\right)^2 + \left(\frac{E_Y}{Y}\right)^2}.$$

Measurement uncertainty propagation



Calculating uncertainty

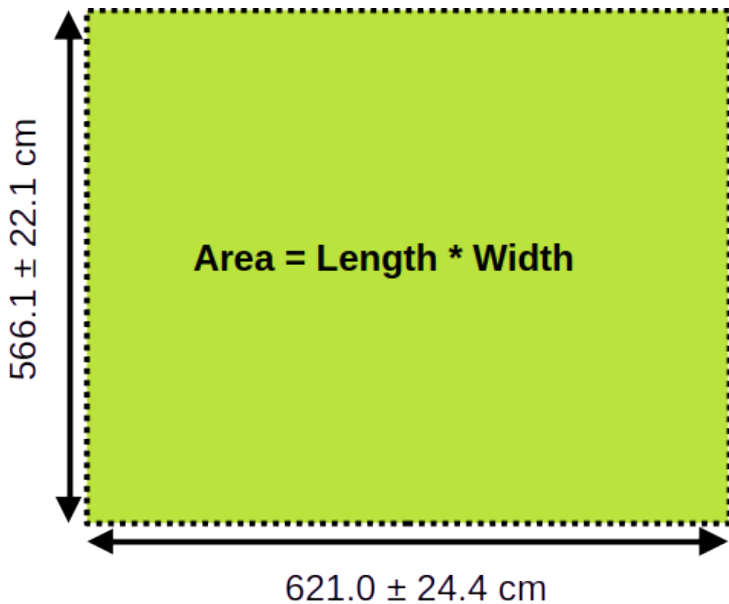
$$E_Z = Z \sqrt{\left(\frac{E_X}{X}\right)^2 + \left(\frac{E_Y}{Y}\right)^2}.$$

- ▶ $X = 432$
- ▶ $Y = 162$
- ▶ $E_X = 12.3$
- ▶ $E_Y = 5.6$

Calculating uncertainty

$$E_Z = Z \sqrt{\left(\frac{E_X}{X}\right)^2 + \left(\frac{E_Y}{Y}\right)^2}.$$

Measurement uncertainty propagation



Calculating uncertainty

$$E_Z = Z \sqrt{\left(\frac{E_X}{X}\right)^2 + \left(\frac{E_Y}{Y}\right)^2}.$$

Working with jamovi

The screenshot displays the Jamovi software interface. The title bar reads "bumpus". The main menu bar includes "Variables", "Data", "Analyses", and "Edit". Below this is a toolbar with icons for "Exploration", "T-Tests", "ANOVA", "Regression", "Frequencies", "Factor", "distrACTION", and "Modules".

The central area shows a data table with the following columns: *surv*, *totlen*, *wingext*, *wgt*, *head*, *humer*, and *femu*. The data is as follows:

	<i>surv</i>	<i>totlen</i>	<i>wingext</i>	<i>wgt</i>	<i>head</i>	<i>humer</i>	<i>femu</i>
1	alive	154	241	24.5	31.2	0.687	
2	alive	160	252	26.9	30.8	0.736	
3	alive	155	243	26.9	30.6	0.733	
4	alive	154	245	24.3	31.7	0.741	
5	alive	156	247	24.1	31.5	0.715	
6	alive	161	253	26.5	31.8	0.780	
7	alive	157	251	24.6	31.1	0.741	
8	alive	159	247	24.2	31.4	0.728	
9	alive	158	247	23.6	29.8	0.703	
10	alive	158	252	26.2	32.0	0.749	
11	alive	160	252	26.2	32.0	0.741	
12	alive	162	253	24.8	32.3	0.766	
13	alive	161	243	25.4	31.8	0.721	
14	alive	160	250	23.7	29.8	0.730	
15	alive	159	247	25.7	31.4	0.729	
16	alive	158	253	25.7	31.9	0.743	
17	alive	159	247	26.5	31.6	0.733	
18	alive	166	253	26.7	32.5	0.767	
19	alive	159	247	23.9	31.4	0.752	
20	alive	160	248	24.7	31.3	0.752	

The status bar at the bottom indicates "Ready", "Filters 0", "Row count 136", "Filtered 0", "Deleted 0", "Added 0", and "Cells edited 0". On the right side, there is a logo and the text "version 2.6.44".

Working with jamovi

The screenshot displays the jamovi software interface. The main window is titled "bumpus" and has a menu bar with "Variables", "Data", "Analyses", and "Edit". Below the menu bar is a toolbar with icons for Exploration, T-Tests, ANOVA, Regression, Frequencies, Factor, and distractionION. A "Modules" button with a plus sign is on the right.

The left pane shows a data table with two columns: "surv" and "totlen". The "surv" column contains values "alive" for rows 1-20, and the "totlen" column contains numerical values ranging from 15 to 16. The "Analyses" menu is open, and the "Descriptives" window is active.

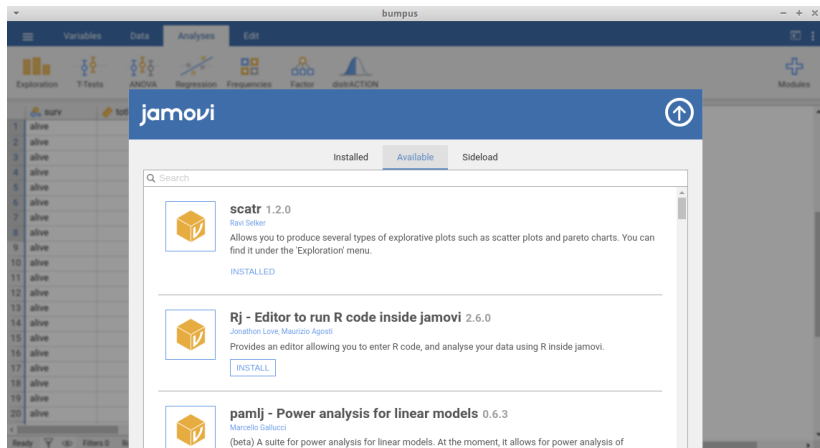
The "Descriptives" window has a search bar and a list of variables: "surv", "wingext", "wgt", "head", "humer", "femur", "tibia", "skull", and "stern". The "totlen" variable is selected in the "Variables" list. The "Split by" list is empty. The "Descriptives" dropdown is set to "Variables across columns". There is a checkbox for "Frequency tables" which is currently unchecked.

At the bottom of the "Descriptives" window, there are expandable sections for "Statistics" and "Plots".

The right pane is titled "Results" and shows the "Descriptives" output for the "totlen" variable. The output is as follows:

Descriptives	
totlen	
N	136
Missing	0
Mean	159.54412
Median	160.00000
Standard deviation	3.56083
Minimum	152
Maximum	167

Working with jamovi



The screenshot displays the jamovi software interface. The main window is titled "bumpus" and features a menu bar with "Variables", "Data", "Analyses", and "Edit". Below the menu bar is a toolbar with icons for "Exploration", "T-Tests", "ANOVA", "Regression", "Frequencies", "Factor", and "distACTION". On the left, a data table is visible with a column labeled "surv" and 20 rows of "alive" entries. The central panel shows the "jamovi" logo and a search bar. Below the search bar are three tabs: "Installed", "Available", and "Sideload". The "Available" tab is selected, showing a list of modules:

- scatr 1.2.0**
Ravi Selker
Allows you to produce several types of explorative plots such as scatter plots and pareto charts. You can find it under the 'Exploration' menu.
[INSTALLED](#)
- Rj - Editor to run R code inside jamovi 2.6.0**
Jonathon Love, Maurizio Agosti
Provides an editor allowing you to enter R code, and analyse your data using R inside jamovi.
[INSTALL](#)
- pamlj - Power analysis for linear models 0.6.3**
Marcello Gallucci
(beta) A suite for power analysis for linear models. At the moment, it allows for power analysis of