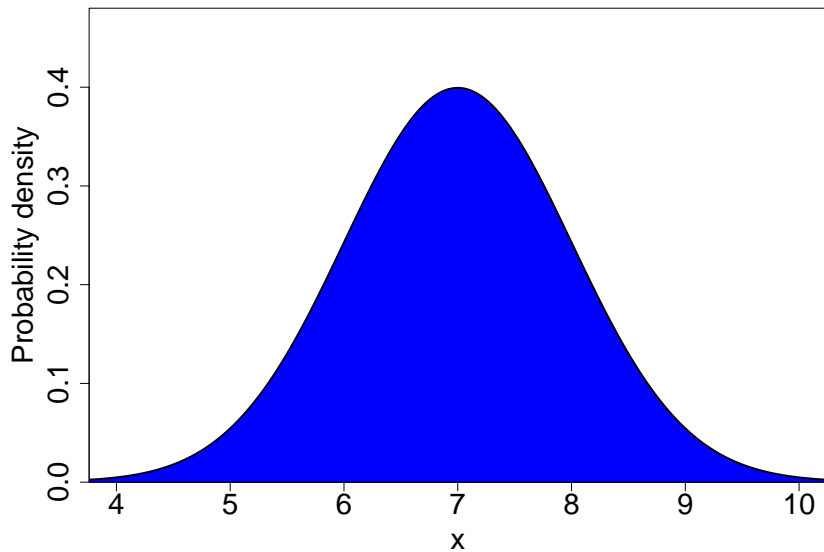
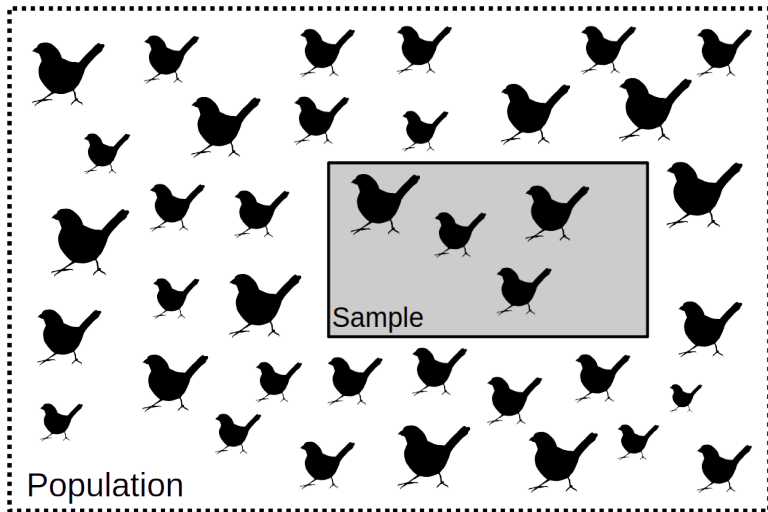


# The Central Limit Theorem (CLT)

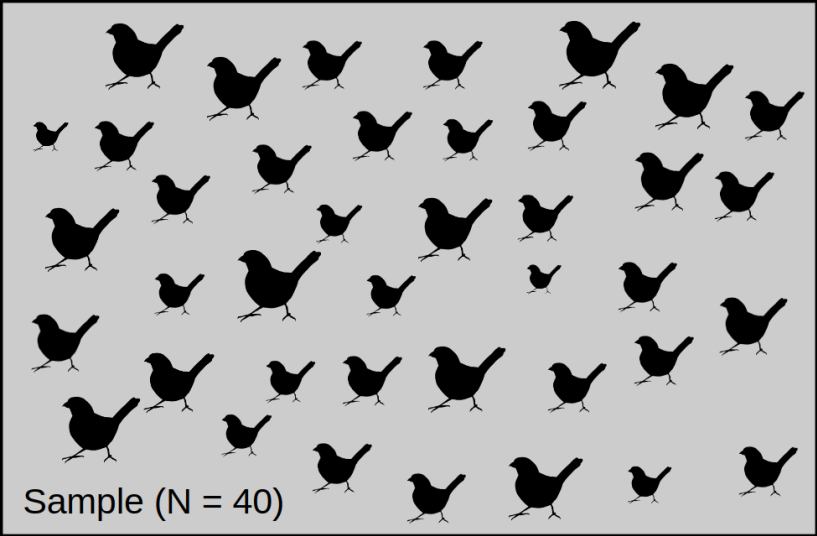
## Normal distribution



# Sampling from a population (distribution)



Consider the distribution of a sample of  $N = 40$



Consider the distribution of a sample of  $N = 40$

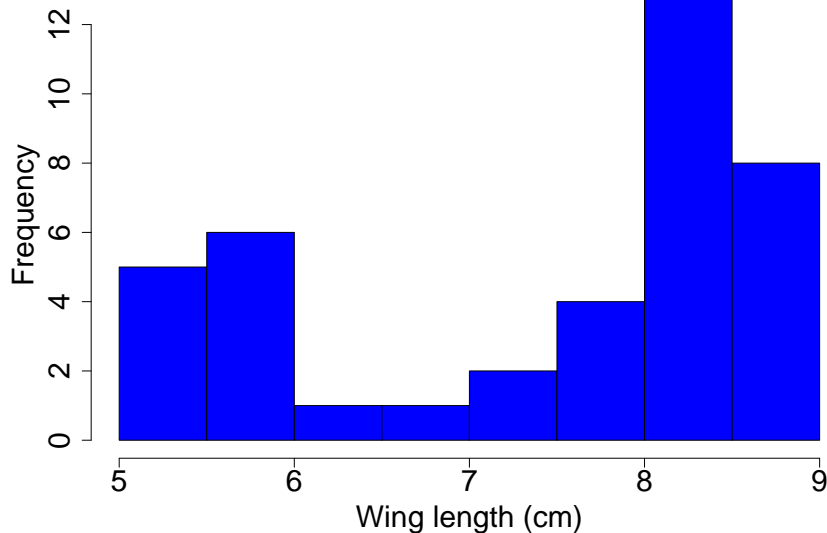
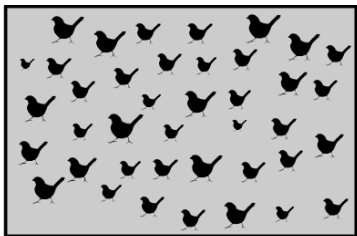
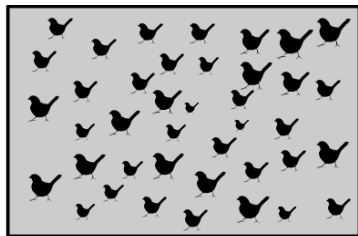


Figure 1: Distribution of a sample of bird wing length (cm)

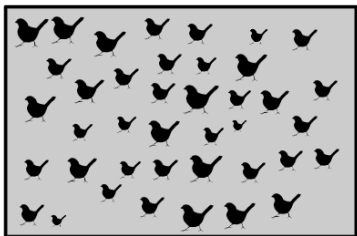
Sample means always distributed normally around true mean



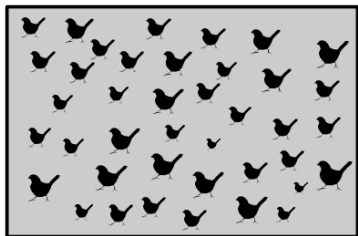
Sample mean = 6.59 cm



Sample mean = 7.45 cm

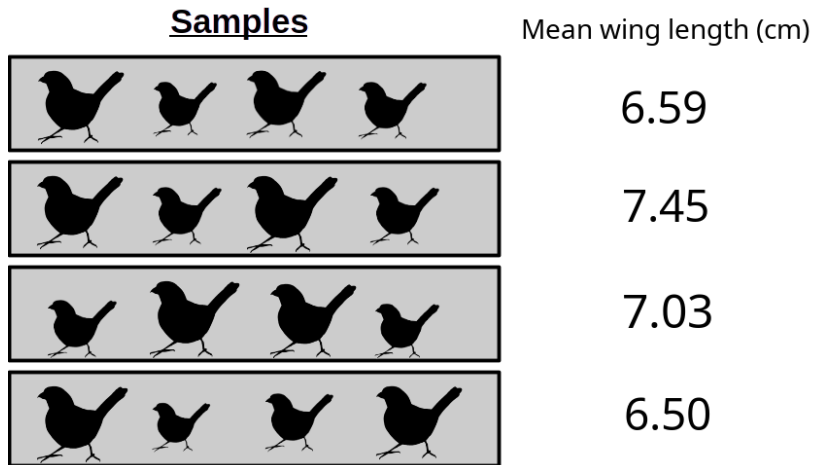


Sample mean = 7.03 cm



Sample mean = 6.50 cm

Sample means always distributed normally around true mean



What if we had 1000 sample means?

Sample means always distributed normally around true mean

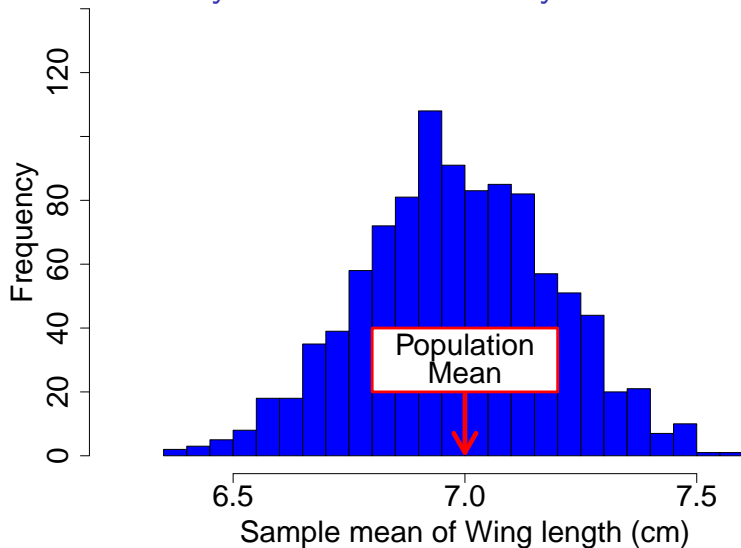


Figure 2: Distribution of the mean of bird wing length (cm) samples ( $N = 40$ ), with sampling repeated 1000 times.

Sample means always distributed normally around true mean

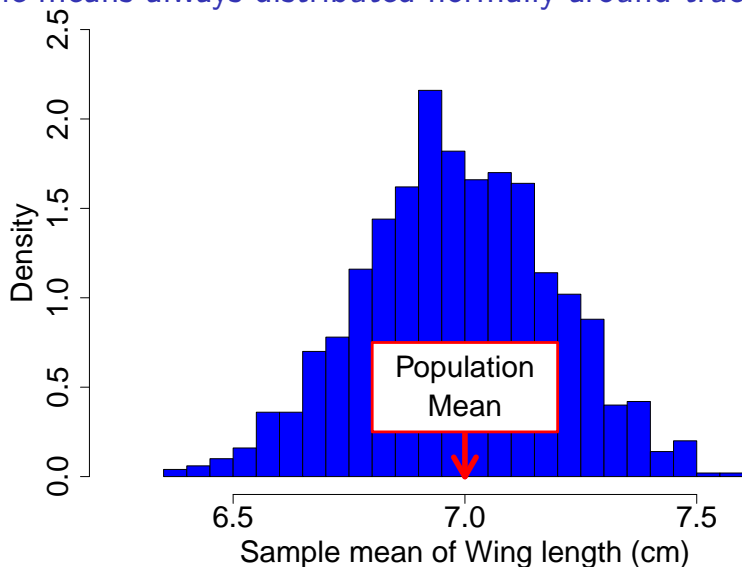


Figure 3: Distribution of the mean of bird wing length (cm) samples ( $N = 40$ ), with sampling repeated 1000 times.

## Sample means always distributed normally around true mean

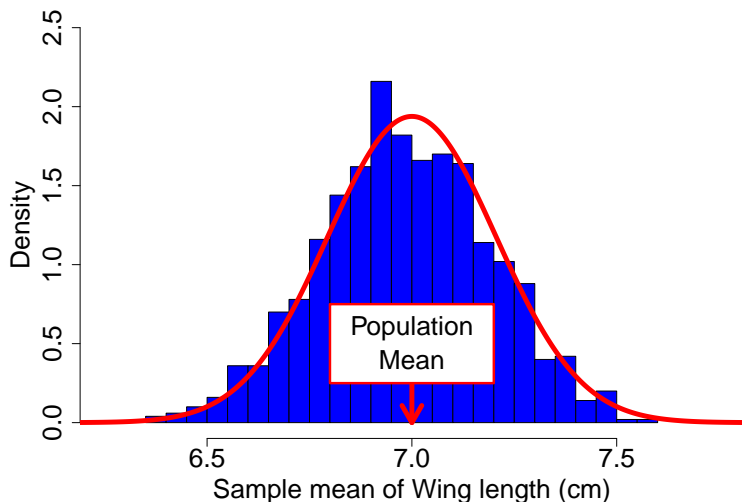
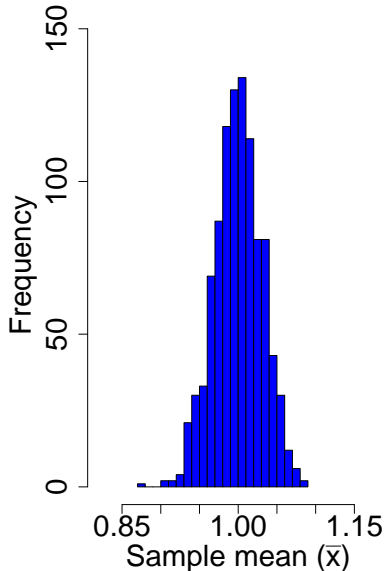
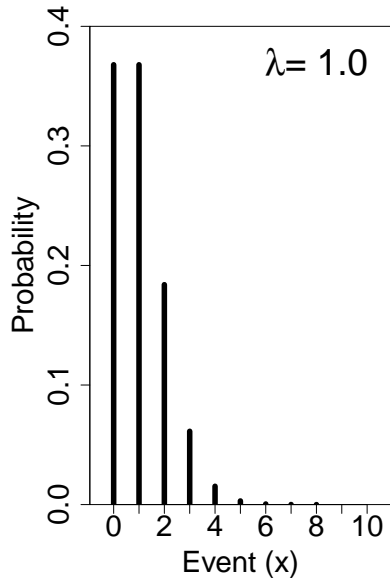


Figure 4: Distribution of the mean of bird wing length (cm) samples ( $N = 40$ ), with sampling repeated 1000 times.

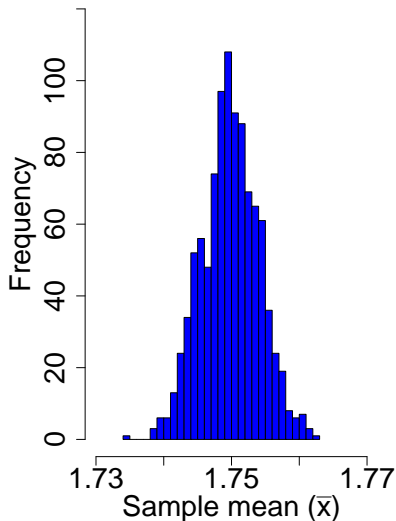
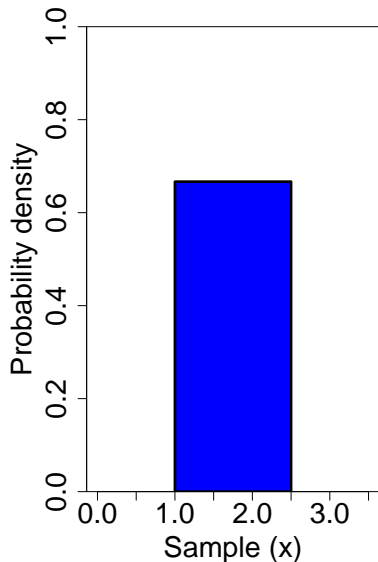
Sample means always distributed normally around true mean

- ▶ Distribution of sample means gets closer to the population mean with increasing  $N$
- ▶ Can demonstrate this ourselves with simulated data

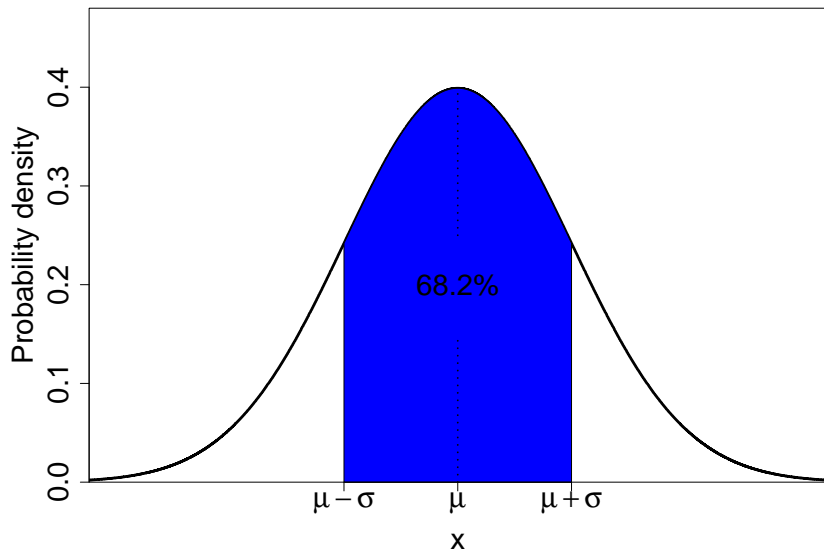
Sample means always distributed normally around true mean



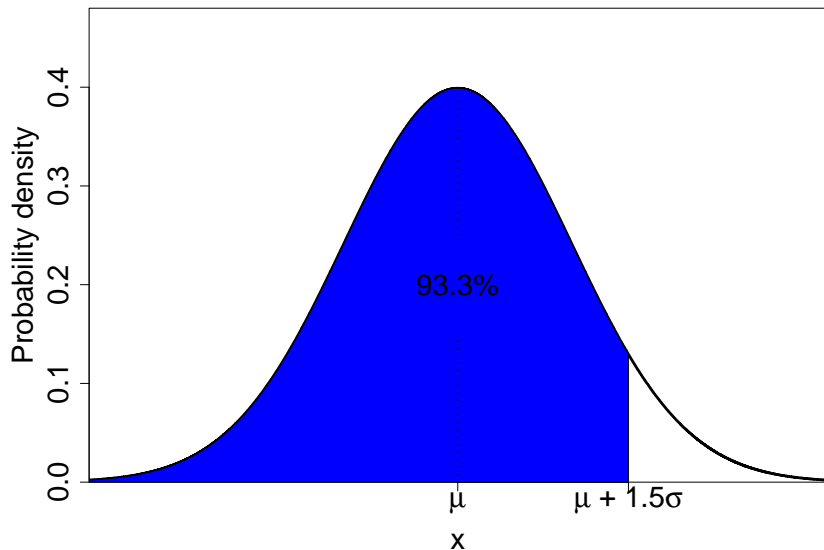
Sample means always distributed normally around true mean



## Probability density and the normal distribution



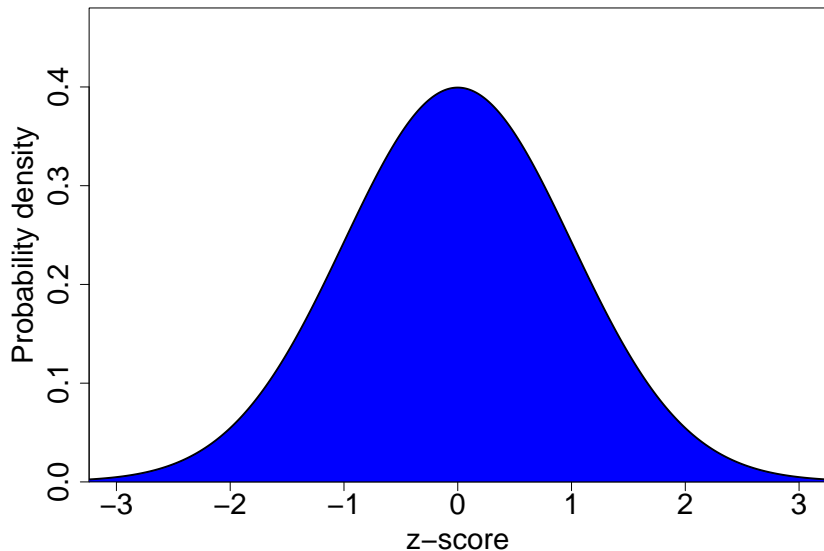
## Probability density and the normal distribution



## Standard normal distribution

- ▶ Need some standard values of  $x$  to work out probability
- ▶ Standard normal distribution
  - ▶ Mean ( $\bar{x}$ ) = 0
  - ▶ Standard deviation ( $s$ ) = 1
- ▶ Baseline for comparison

## Standard normal distribution



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<sup>1</sup><https://bradduthie.github.io/stats/app/zandp/>

## Standard normal distribution

- ▶ **z-score**: deviation from 0
- ▶ Work out probability density
- ▶ Basis for confidence intervals
- ▶ Critical for hypothesis testing

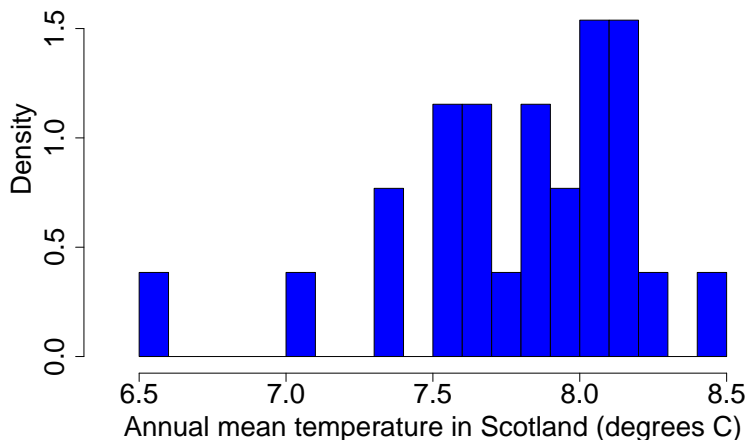
## Translating a variable into a standard normal distribution

- ▶ Subtract mean
- ▶ Divide by standard deviation

$$z = \frac{x - \bar{x}}{s}$$

Can turn x-values into z-scores

## Turning a value into z-score

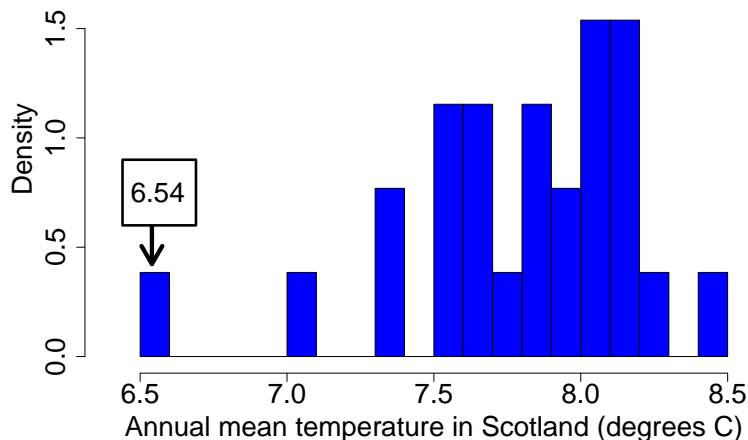


$$N = 26 \quad \bar{x} = 7.789 \quad s = 0.4124844$$

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<sup>1</sup>Met Office. (July 1, 2021). Annual mean temperature in Scotland from 1995 to 2020. In Statista. Retrieved February 10, 2026, from <https://www.statista.com/statistics/367853/scotland-average-temperature/>

## Turning a value into z-score



$$N = 26 \quad \bar{x} = 7.789 \quad s = 0.4124844$$

<sup>1</sup>Met Office. (July 1, 2021). Annual mean temperature in Scotland from 1995 to 2020. In Statista. Retrieved February 10, 2026, from <https://www.statista.com/statistics/367853/scotland-average-temperature/>

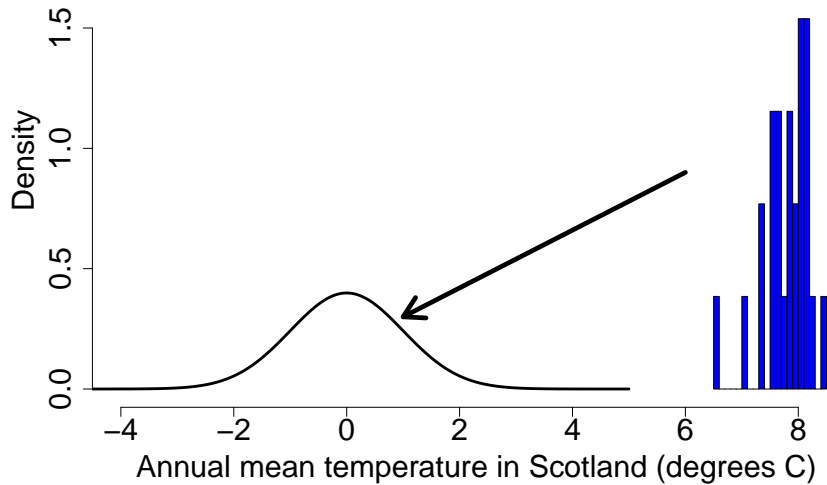
## Turning a value into z-score

- ▶ Subtract mean
- ▶ Divide by standard deviation

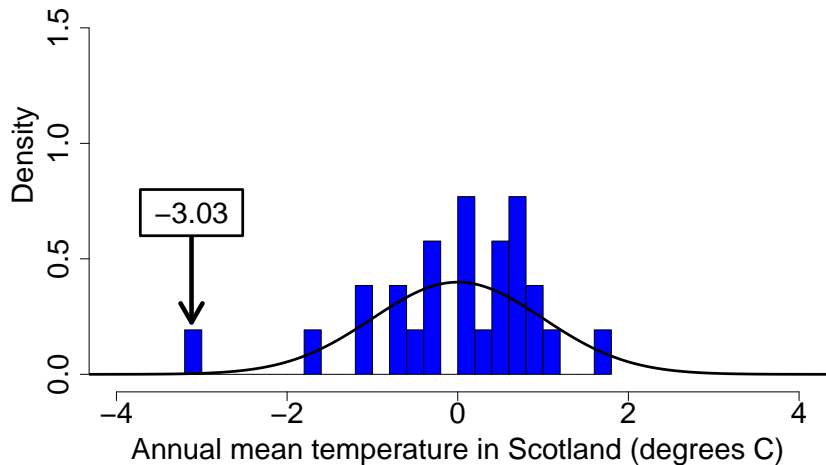
$$z = \frac{6.54 - 7.789}{0.4124844} = -3.03$$

Over 3 standard deviations  
below the mean

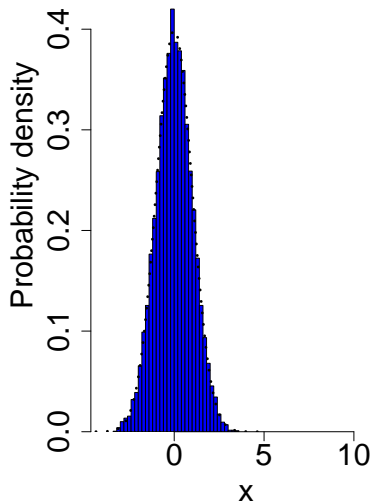
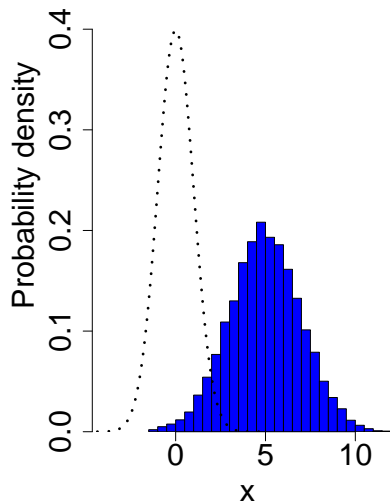
## Turning a value into z-score



## Turning a value into z-score



## Turning a value into z-score



# Exploring distributions in jamovi with distrACTION module

Normal Distribution

Parameters

Mean = 0  
SD = 1

Function

Compute probability  Compute quantile(s)

$P(X \leq x_1)$   cumulative quantile  
  $P(X \geq x_1)$   central interval quantiles  
  $P(x_1 \leq X \leq x_2)$

$x_1 = -3.03$   $p = 0.5$   
 $x_2 = 1$

Normal Distribution

Input values

Parameters	Compute probability
Mean = 0	$x_1 = -3.03$
SD = 1	Mode: $P(X \leq x_1)$

Results

Probability
0.00122

0.4  
0.3  
0.2  
0.1  
0.0

-4 -3 -2 -1 0 1 2 3 4

P (Area)