Regression

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Regression analysis can be used to support causal hypotheses, but it **cannot, by itself be used to determine causality.**

Visualising a regression of y values against x values

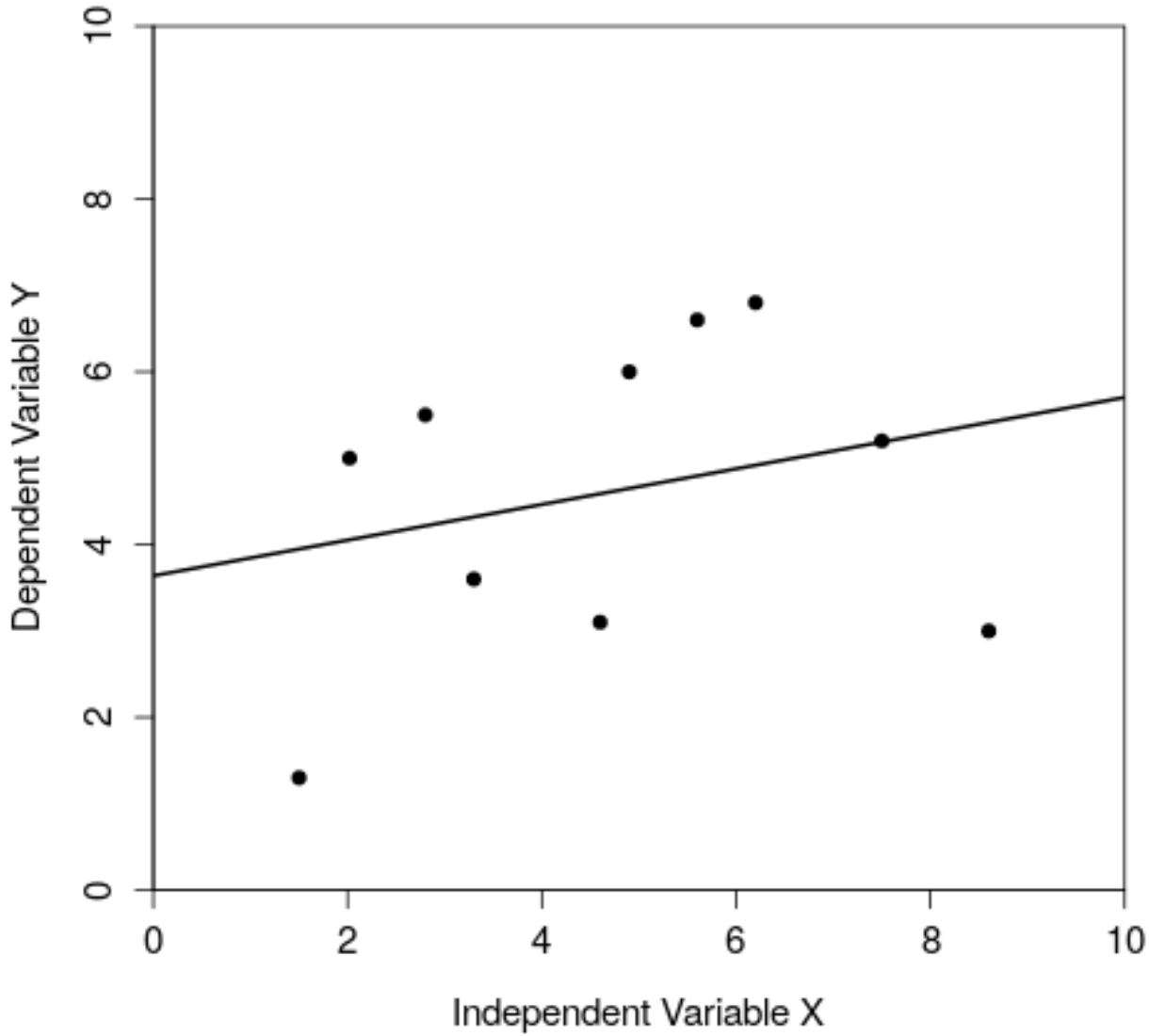

Figure 1: Regression of dependent variable y against independent x.

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In an experiment, the independent variable is something that we as researchers have control over (e.g., amount of fertiliser to put down on a field), whereas the dependent variable is something that we would measure when collecting our data (e.g., total crop yield of the field).

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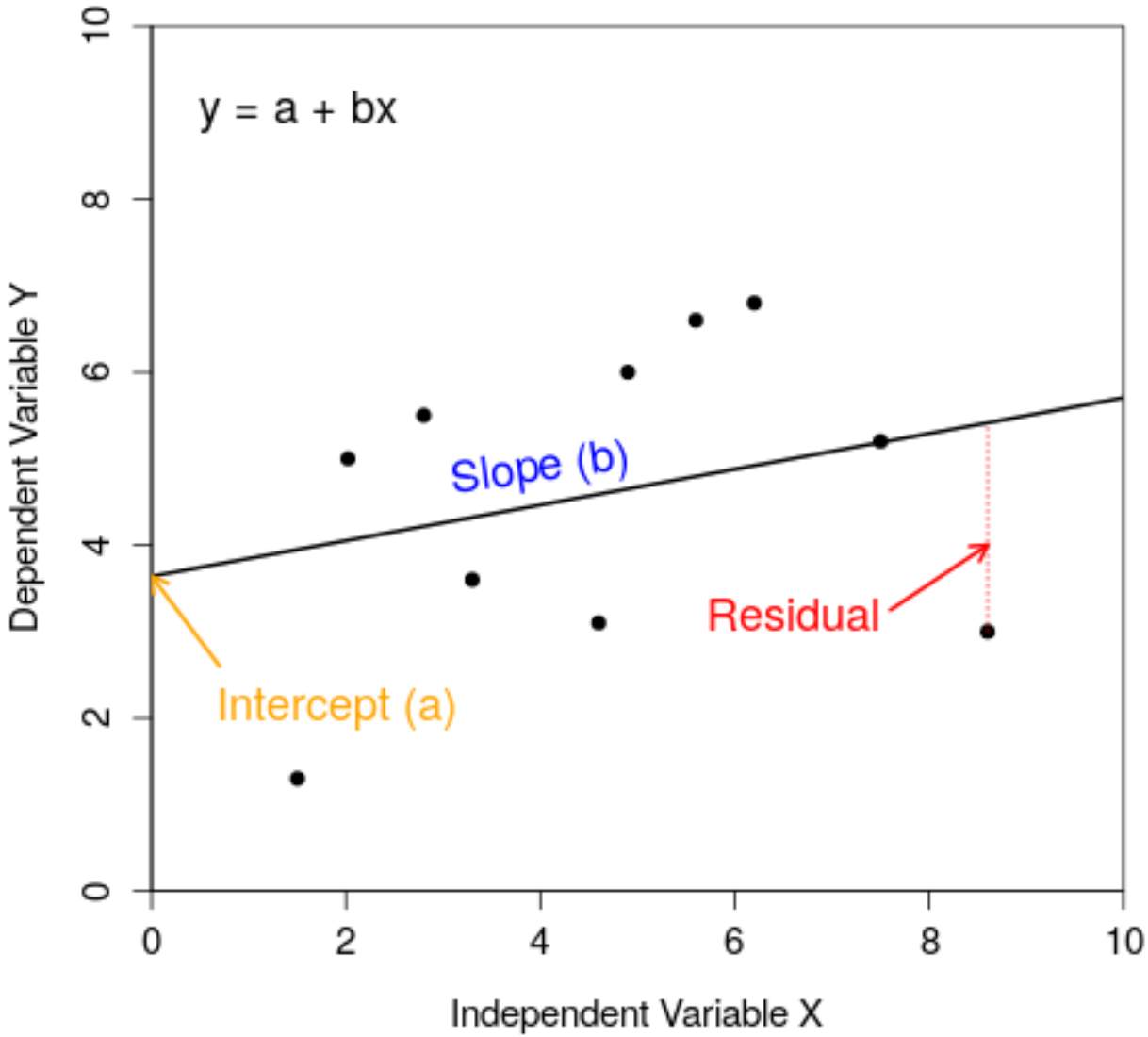
Note that data points rarely will sit right on the regression line. The **residual** is defined by the difference between the measured value of y (i.e., the data point) and the y value predicted by the regression line (i.e., the vertical distance between the data point and the line).

Regression line



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To get an intuitive sense for how the regression line minimises the sum of squares, use **[this interactive application]** to adjust the slope and intercept to try to find the line of best fit (it will turn blue when you succeed).

Assumptions of regression

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Note that even if our assumptions are not perfectly met (indeed, they rarely if ever will be), this does not completely invalidate the method of linear regression. But large violations of one or more of these assumptions might indeed be problematic.

Assumptions of regression

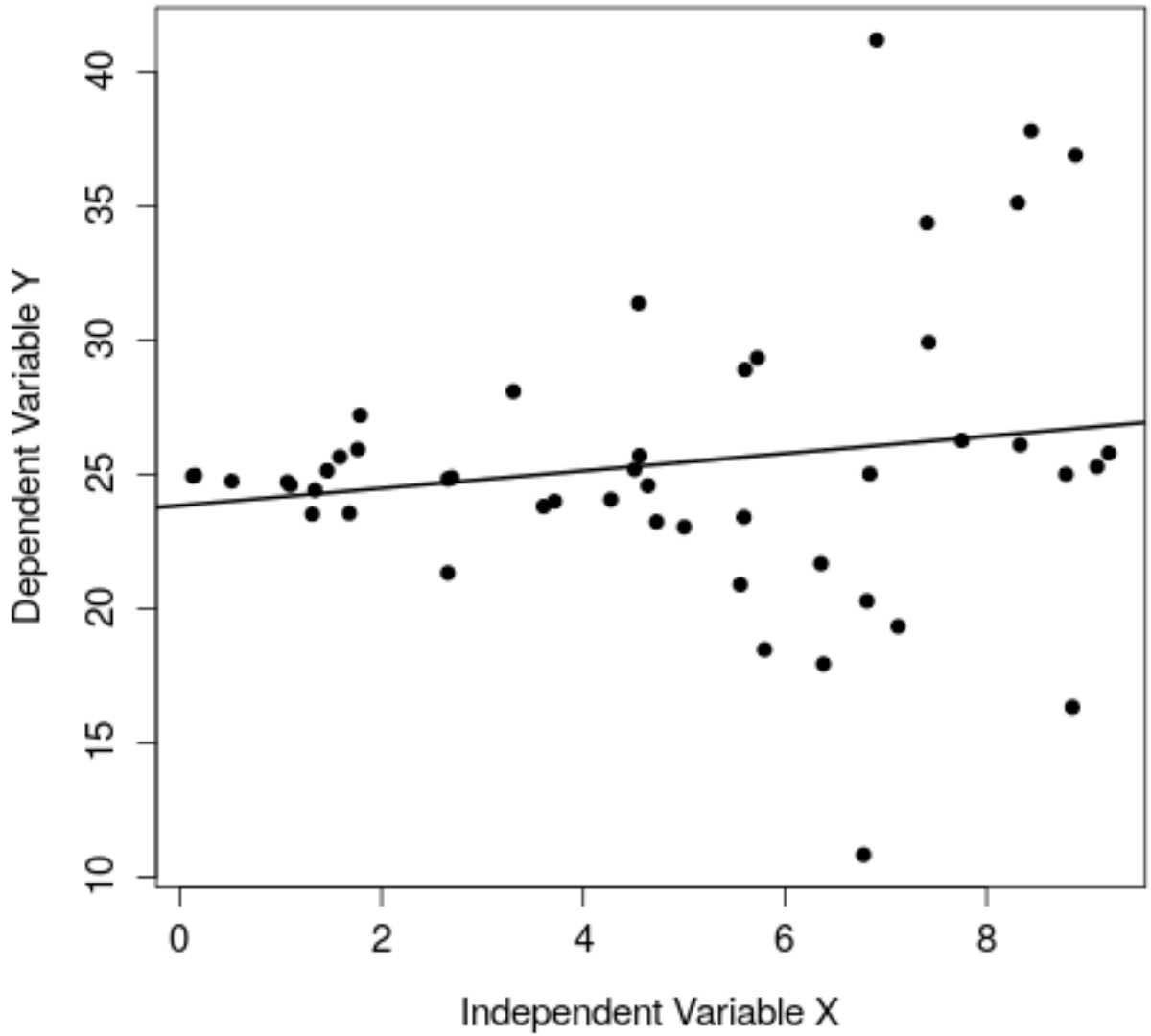

Figure 3: A regression of one dependent variable y against the independent

How good is the fit of our model?

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See the examples below of four different R^2 values to see what this looks like.

Scatterplots of different coefficients of determination

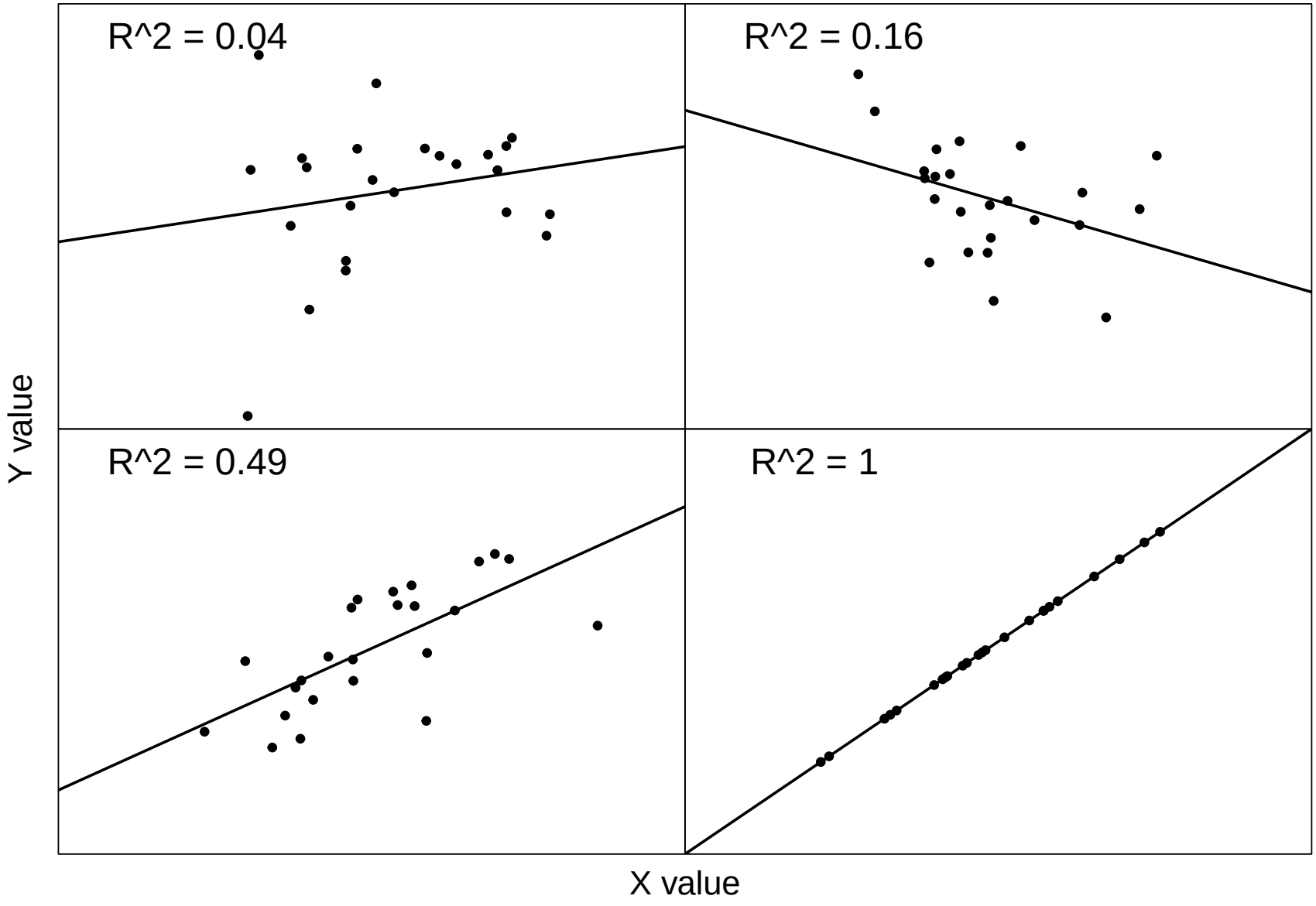

Figure 4: Four different coefficients of determination

More understand of the coefficient of determination

Understanding that **the coefficient of determination tells us how much variation in y is explained by the regression equation** is the important point.

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- Coefficient of determination compares the sum of squared residuals from the linear model (SS_{res}) to what the sum of squared residuals would be if we had just use the mean value of y (SS_{tot}).
- Conveniently, R^2 is also just the Pearson product moment correlation (r) squared.

Visualising the coefficient of determination

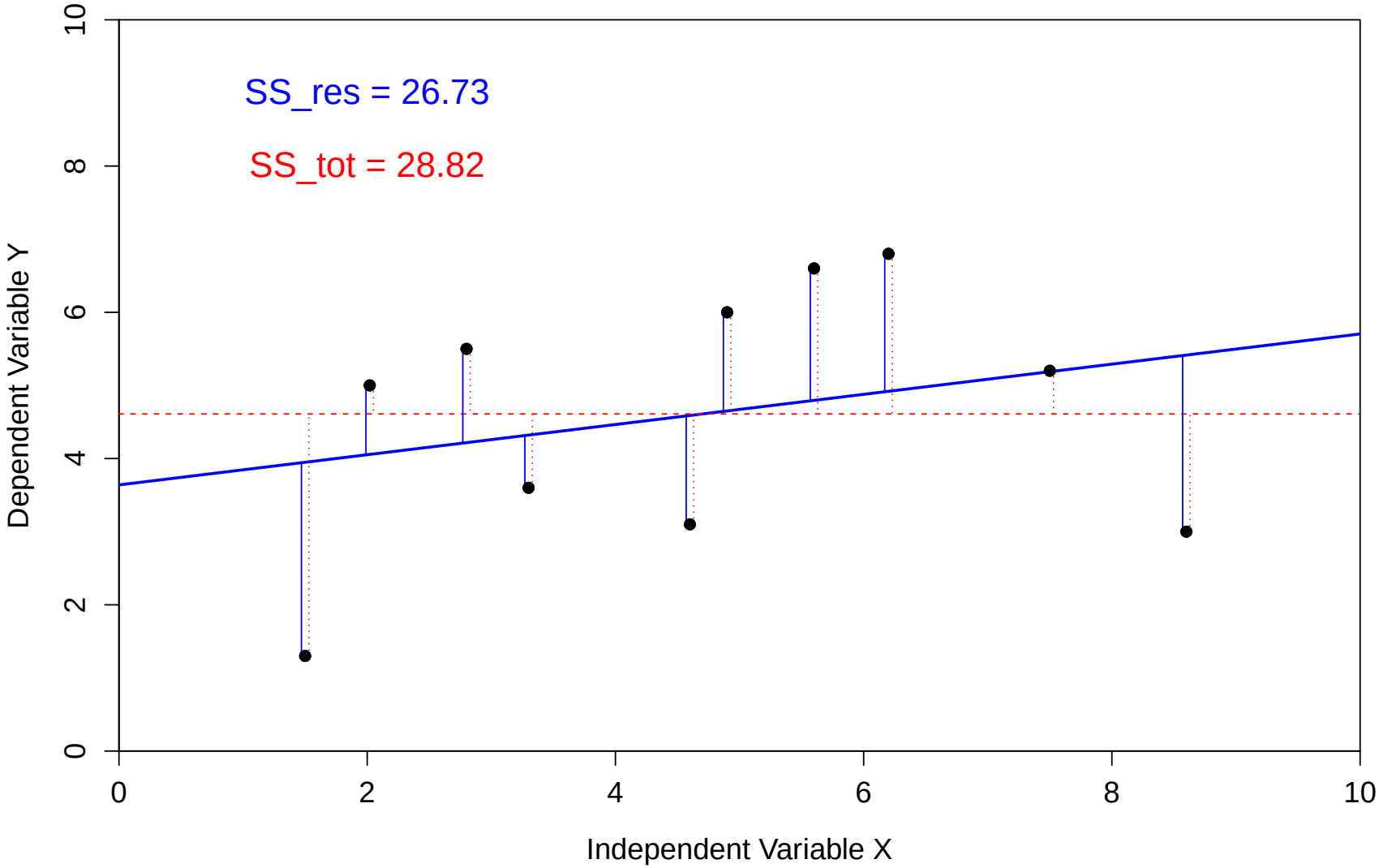

Figure 5: A regression of one dependent variable y against the independent variable x. Blue vertical lines show residuals of the linear model, while red dotted vertical lines show residual deviations of from the mean of y.

The F-test of overall significance

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Hypothesis for F-test of overall significance of a linear model

- **Null:** The model with no independent variables fits the data as well as the linear model
- **Alternative:** The linear model fits the data better than the model with no independent variables

Understand is how to interpret the F-test of overall significance (see below for doing this in practice).

Significance of equation parameters

- Test the significance of individual parameters in the linear model (β_0 and β_1; recall that $y = \beta_0 + \beta_1 x$).
- For both coefficients β_0 and β_1, we can state the null and alternative hypotheses

Hypothesis for coefficient of a linear model

- **Null:** The value of the coefficient equals 0.
- **Alternative:** The value of the coefficient does not equal 0.

Statistical software such as jamovi will calculate p-values to test our null hypothesis for both β_0 and β_1 coefficients.

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4. Ideally, test the regression model on new observational data to examine how close the predicted values are to the observations

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Care should be taken when extrapolating beyond the measured data because the relationship between the two variables might change.

Predictions with linear regression

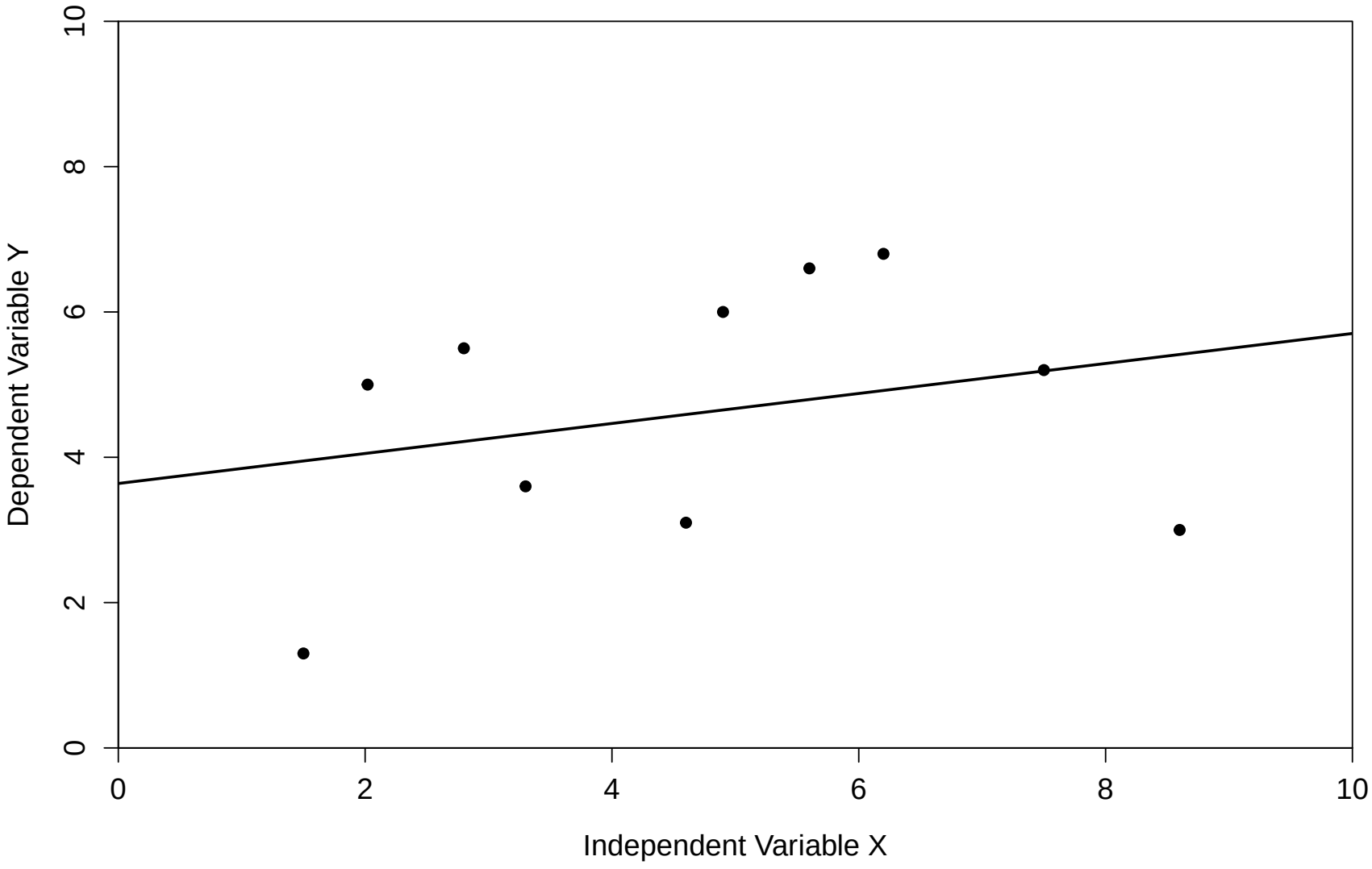

Figure 6: Regression of dependent variable y against independent x.