SCIU4T4: Central tendency and measures of spread

Descriptive versus inferential statistics

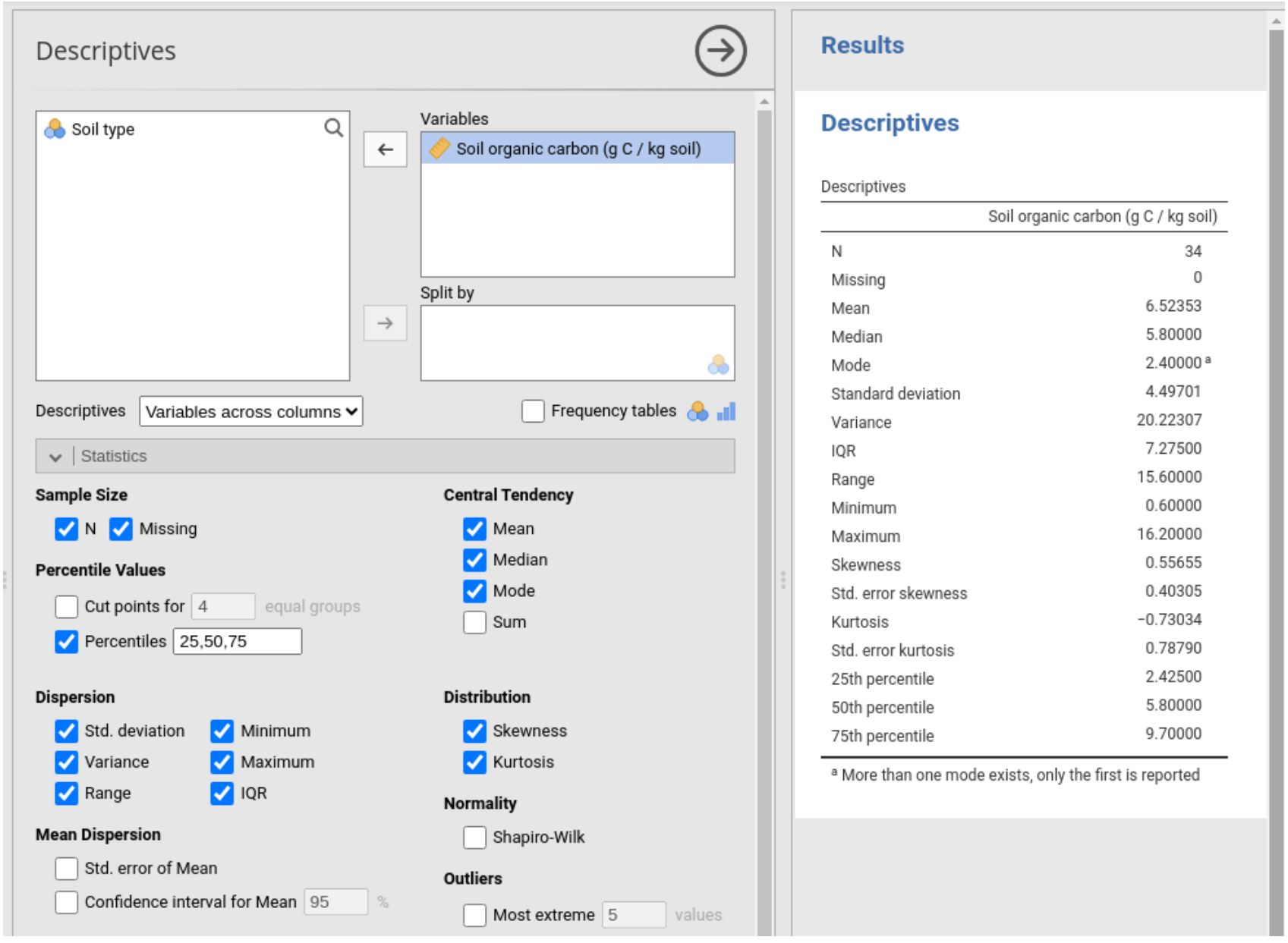
Descriptive Statistics

- Summarise observations
- E.g., average monthly temperature

Inferential Statistics

- Make estimates or predictions
- E.g., predict temperature from latitude

Descriptive statistics in jamovi

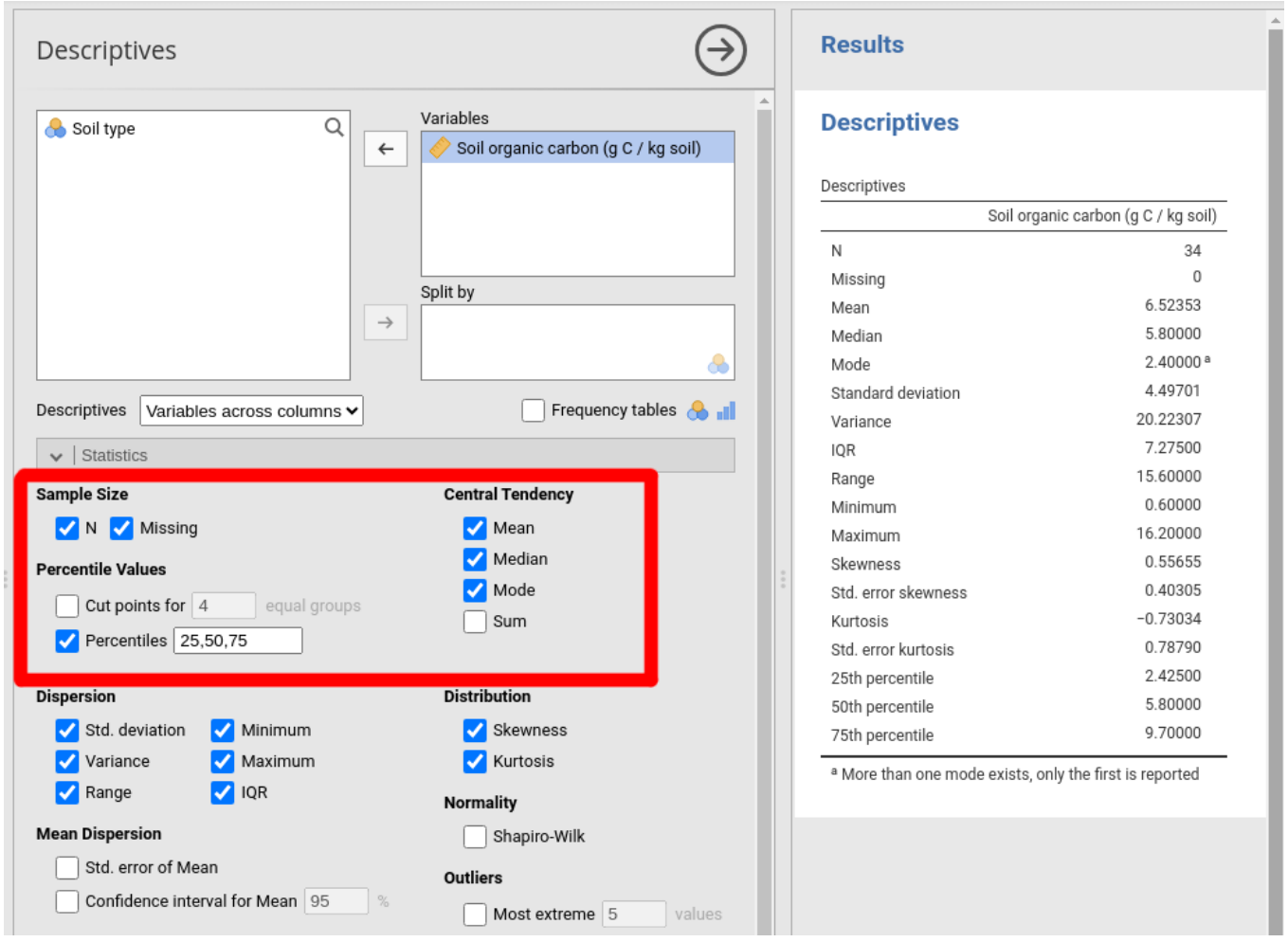


Properties of distributions

- Central tendency
- Spread
- Skew & Kurtosis

Will focus on *samples* rather than populations

Descriptive statistics: Central tendency



Mean, median, and mode

Arithmetic mean

Add values, divide by number (N)

For example, $N = 3$ temperatures:

- $12.5\,^{\circ}C$
- $13.4\,^{\circ}C$
- $14.0\,^{\circ}C$

$$\bar{x} = \frac{12.5 + 13.4 + 14.0}{3} = 13.3$$

Calculating the mean of 7 temperatures ($^{\circ}C$)

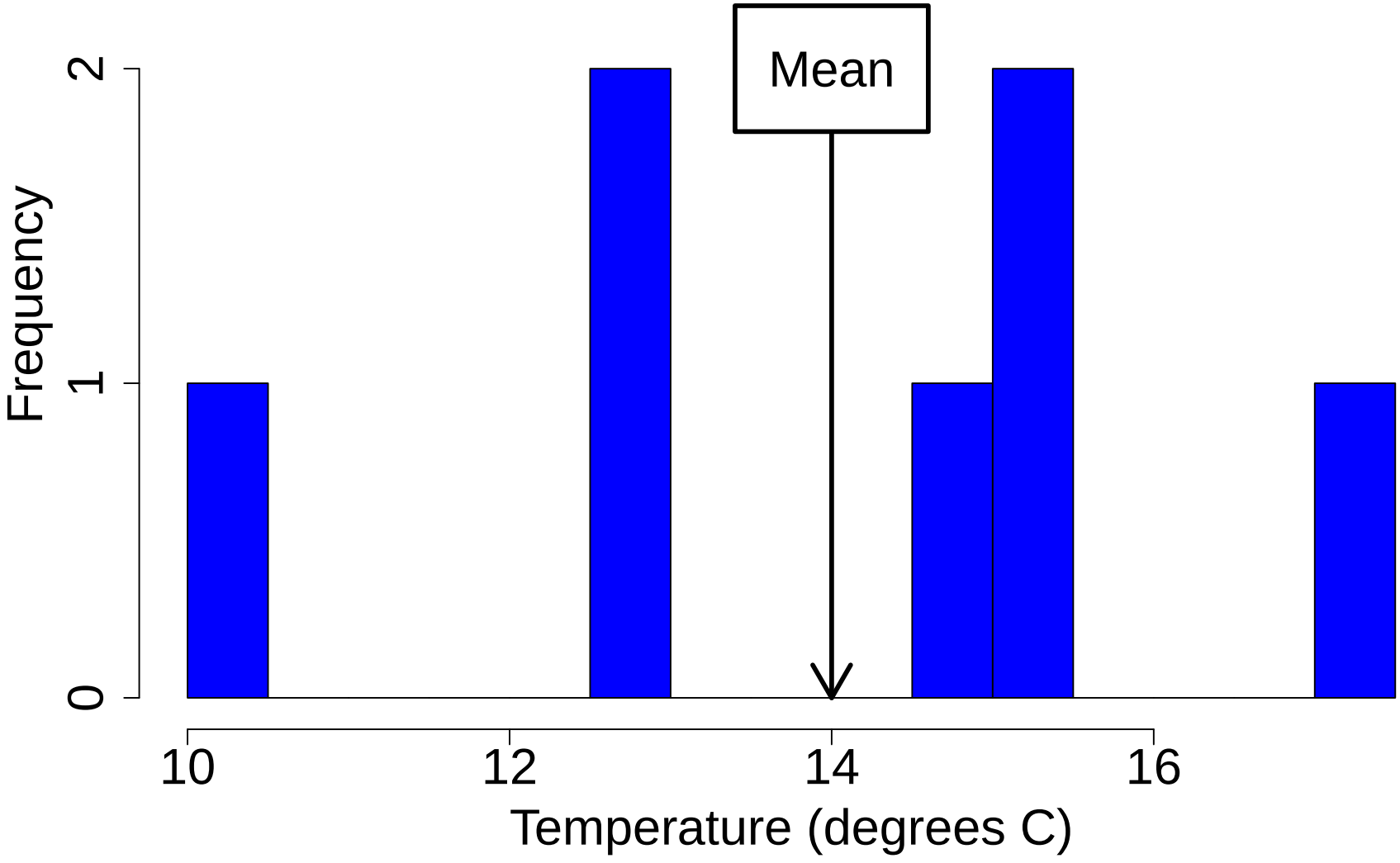
Table 1: Seven values (x) of soil temperature ($^{\circ}C$) at a site

x_1	x_2	x_3	x_4	x_5	x_6	x_7
17.1	15.2	14.9	12.6	15.2	10.3	12.7

$$\bar{x} = \frac{17.1 + 15.2 + 14.9 + 12.6 + 15.2 + 10.3 + 12.7}{7}$$

$$\bar{x} = 14$$

Arithmetic mean visualisation (histogram)



General formula for arithmetic mean

- Sample mean: $\bar{x}$ (or $\hat{\mu}_x$)
- Sample size: N

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_{N-1} + x_N}{N}$$

General formula for arithmetic mean

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_{N-1} + x_N}{N}$$

$$\sum_{i=1}^{N} x_i = x_1 + x_2 + \ldots + x_{N-1} + x_N$$

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

The mode

Most frequently occurring observation

x_1	x_2	x_3	x_4	x_5	x_6	x_7
17.1	**15.2**	14.9	12.6	**15.2**	10.3	12.7

Also applies to categorical data

x_1	x_2	x_3	x_4	x_5	x_6
dog	**cat**	bird	**cat**	**cat**	dog

Visualising the mode

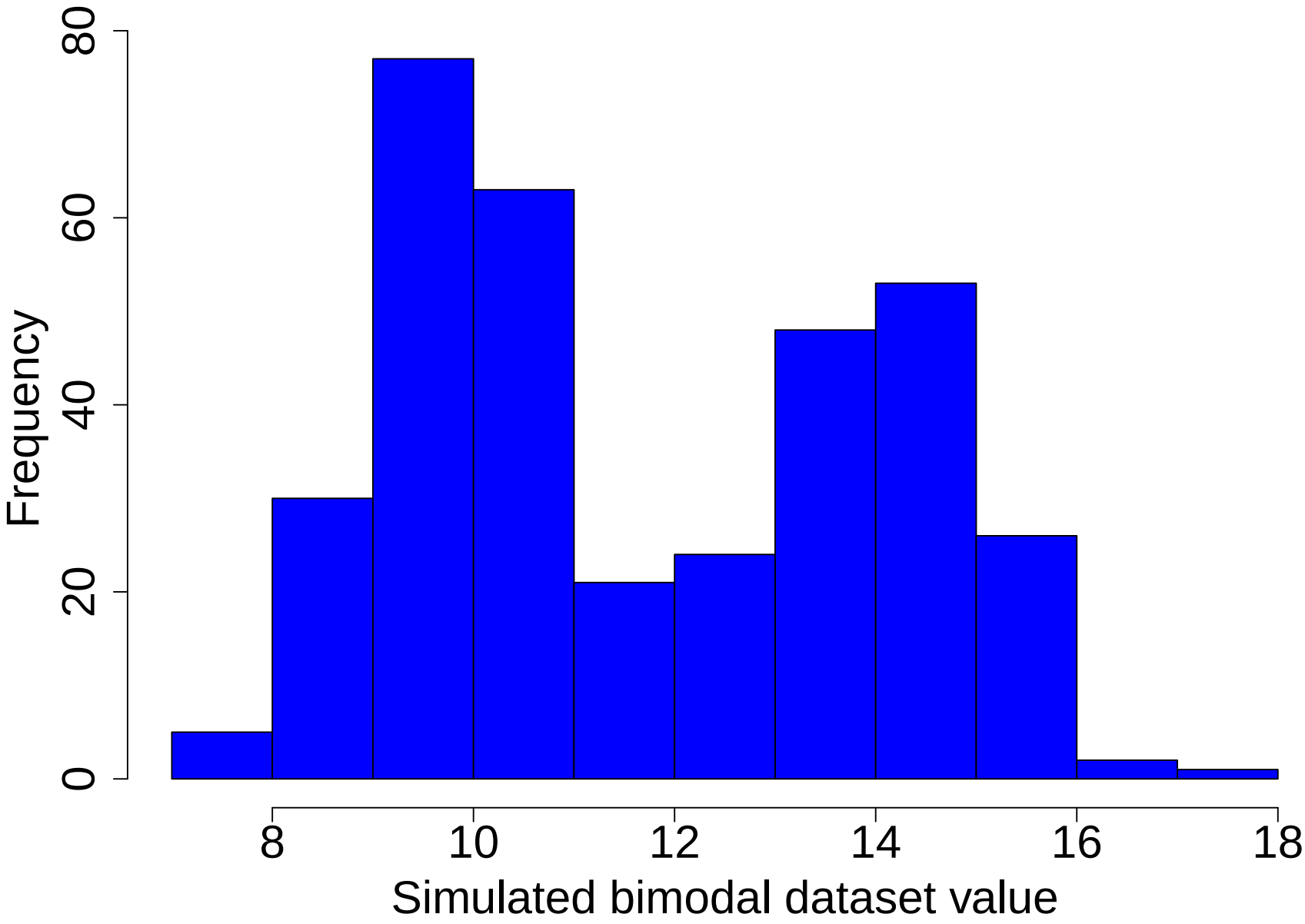


Figure 1: Hypothetical dataset that has a bimodal distribution.

The median

- Observation in the middle when the observations are arranged in ascending order
- There are an equal number of observations lower and higher than the median

The median

x_1	x_2	x_3	x_4	x_5	x_6	x_7
17.1	15.2	14.9	12.6	15.2	10.3	12.7

Sorting the data:

x_6	x_4	x_7	x_3	x_2	x_5	x_1
10.3	12.6	12.7	**14.9**	15.2	15.2	17.1

The median

Median is a type of **quantile** (50%)

- Can break distribution into other quantiles
  - First **quartile** (25% quantile)
  - Third **quartile** (75% quantile)
- Quantiles also called 'percentiles'

x_1	x_2	x_3	x_4	x_5
2	4	5	6	8

The median

If there is no middle value

x_1	x_2	x_3	x_4	x_5	x_6
3.1	3.5	3.8	4.0	4.2	4.2

Take mean of middle values:

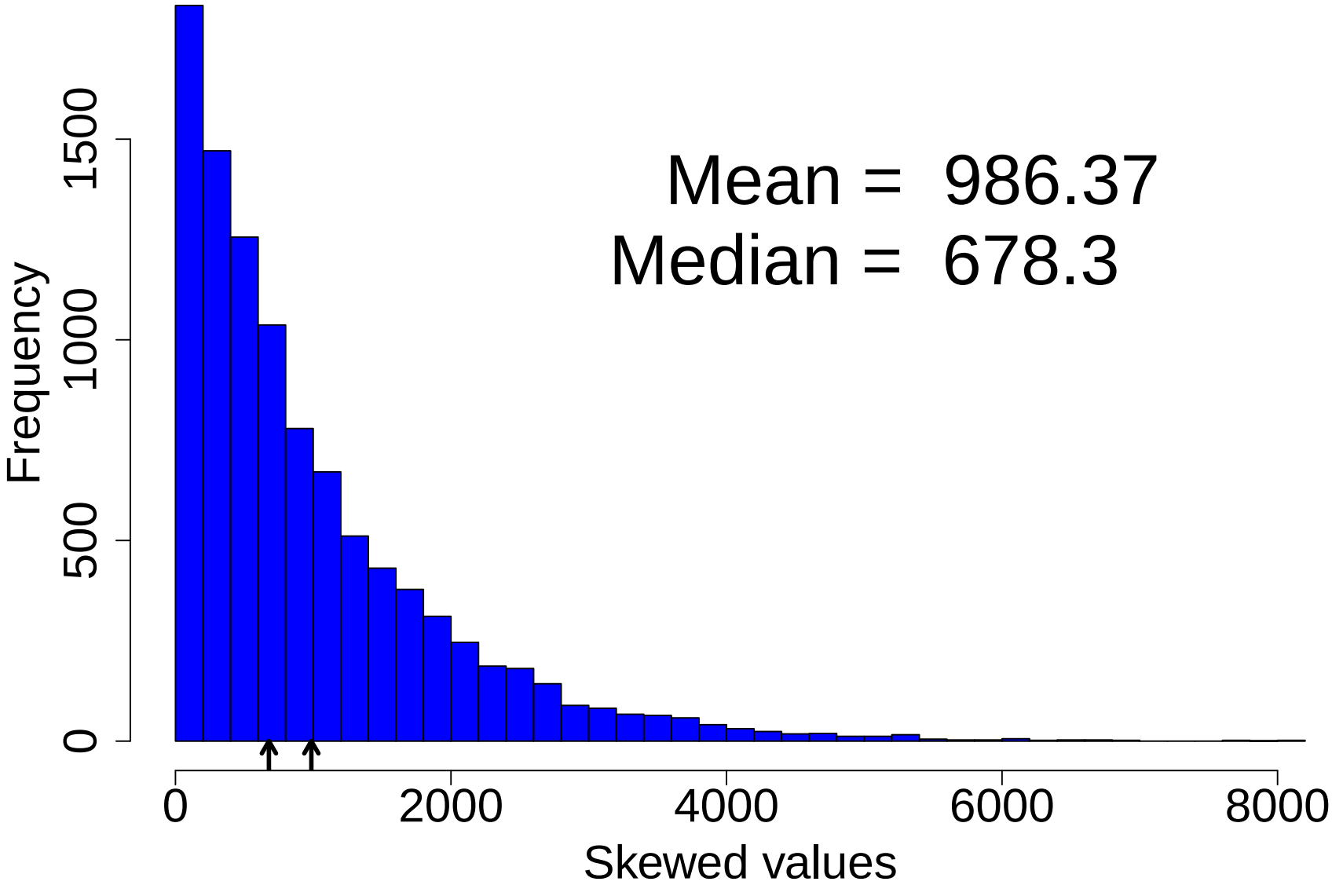
$$\frac{3.8 + 4.0}{2} = 3.9$$

The median

- Multiple valid ways to calculate quantiles[1]
- No one 'right' way
- Jamovi's approach might differ from other software

[1] Hyndman, RJ, & Y Fan. 1996. American Statistician 50:361–65.

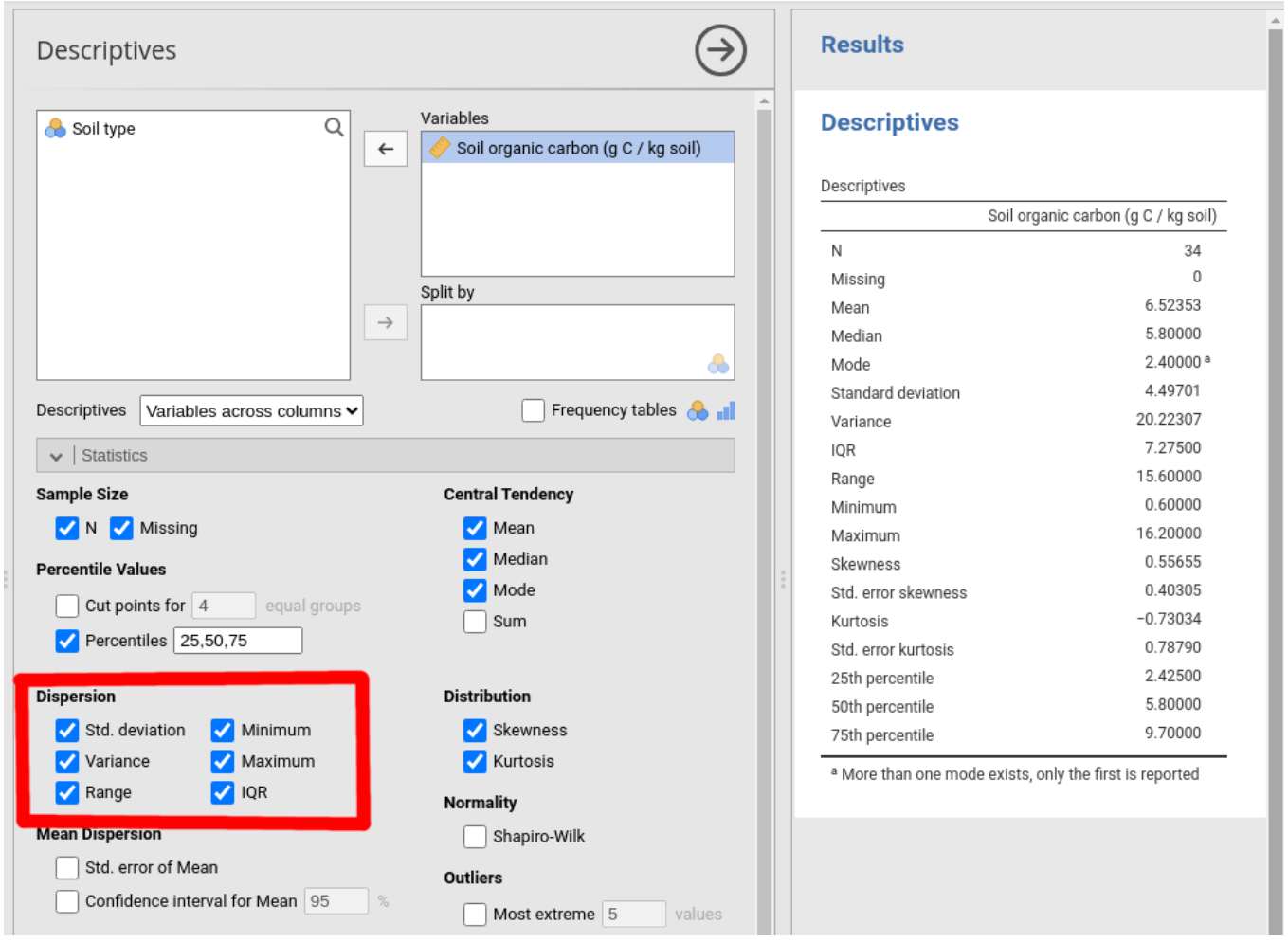
Median more robust to outliers



Measures of spread

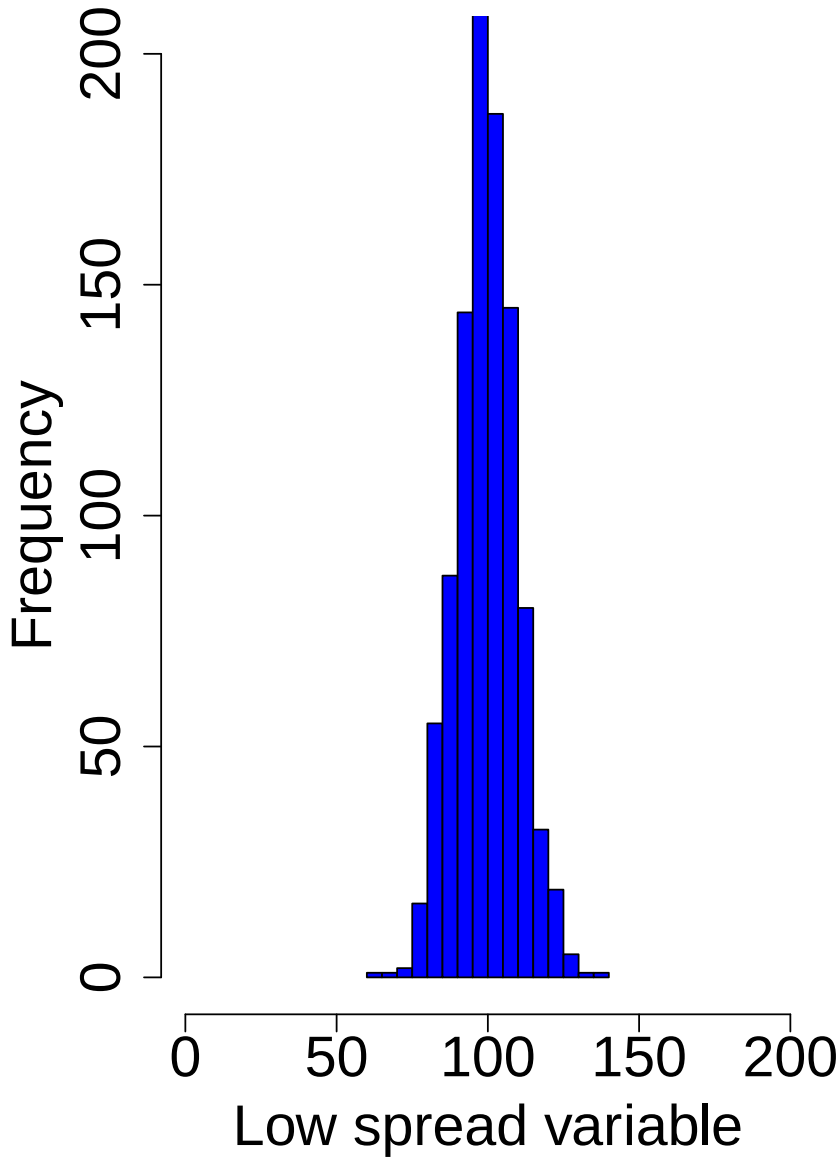
- Range
- Interquartile range (IQR)
- Variance (s^2)
- Standard deviation (s)
- Coefficient of variation (CV)

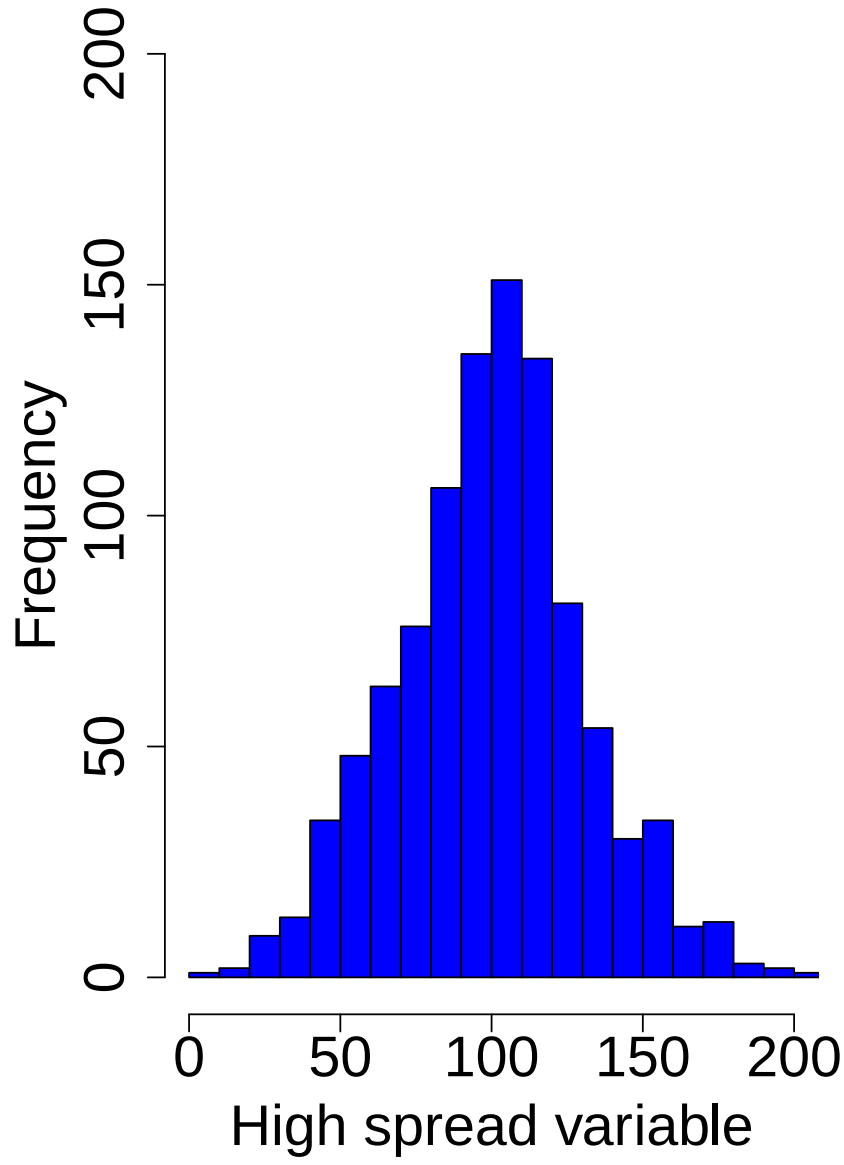
Measures of spread



Range, IQR, s^2, s, CV

Measures of spread





Measures of spread: Range

$$Range(X) = Maximum(X) - Minimum(X)$$

x_1	x_2	x_3	x_4	x_5	x_6	x_7
17.1	15.2	14.9	12.6	15.2	**10.3**	12.7

$$Range(X) = 17.1 - 10.3 = 6.8$$

Measures of spread: Interquartile Range

$$IQR(X) = Q_3(X) - Q_1(X)$$

x_1	x_2	x_3	x_4	x_5
2	**4**	5	**6**	8

$$IQR(X) = 6 - 4 = 2$$

Measures of spread: Variance (s^2)

- Expected squared deviation from mean
- More useful than range or IQR
- Less intuitive than range or IQR[1]
- Jamovi will calculate this for us

$$s^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2 .$$

We can break this down step by step!

[1] https://bradduthie.github.io/stats/app/forest/

Measures of spread: Variance (s^2)

$$s^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2 .$$

x_1	x_2	x_3	x_4	x_5	x_6	x_7
17.1	15.2	14.9	12.6	15.2	10.3	12.7

1. Take x_1 minus mean, squared $(17.1 - 14)^2 = 9.61$
2. Repeat step 1 for $x_2, x_3, ..., x_N$
3. Sum up all these $(x_i - \bar{x})^2$ values
4. Multiply the sum by $1/(N-1)$

Measures of spread: Variance (s^2)

$$s^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2 .$$

x_1	x_2	x_3	x_4	x_5	x_6	x_7
17.1	15.2	14.9	12.6	15.2	10.3	12.7

$$\begin{aligned} SS &= (17.1 - 14)^2 + (15.2 - 14)^2 + \cdots + (12.7 - 14)^2 \\ &= (3.1)^2 + (1.2)^2 + \cdots + (-1.3)^2 \\ &= 30.64 \end{aligned}$$

$$s^2 = \frac{1}{7-1} \times 30.64 = 5.1067 \ ^\circ C^2$$

Measures of spread: Standard deviation (s)

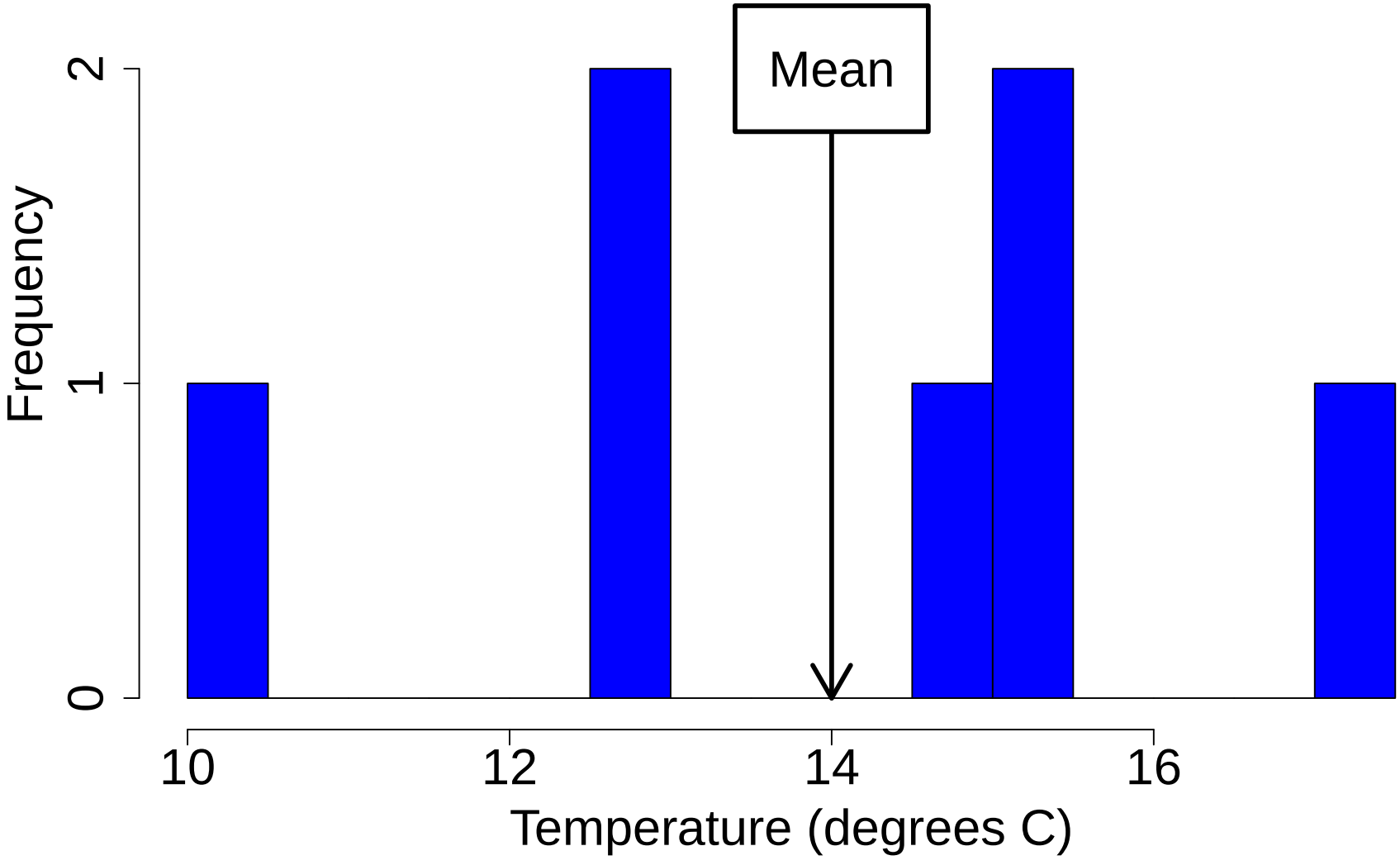
- Mean deviation from the mean
- Square-root of the variance
- Gets back to original units

$$s^2 = 5.1067\ ^\circ C^2$$

$$s = \sqrt{5.1067} = 2.2598\ ^\circ C$$

[2] https://bradduthie.github.io/stats/app/normal_pos_neg/

Standard deviation of the mean: does it look right?



Standard deviation of the mean

$$s = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2}.$$

- One checkbox in jamovi
- Spread of a variable

Coefficient of variation (CV)

Standard deviation divided by the mean

x_1	x_2	x_3	x_4	x_5	x_6	x_7
17.1	15.2	14.9	12.6	15.2	10.3	12.7

$$CV = \frac{s}{\bar{x}} = \frac{2.2598\,^{\circ}C}{14\,^{\circ}C} = 0.1614$$

Note that the units cancel out.

Coefficient of variation (CV)

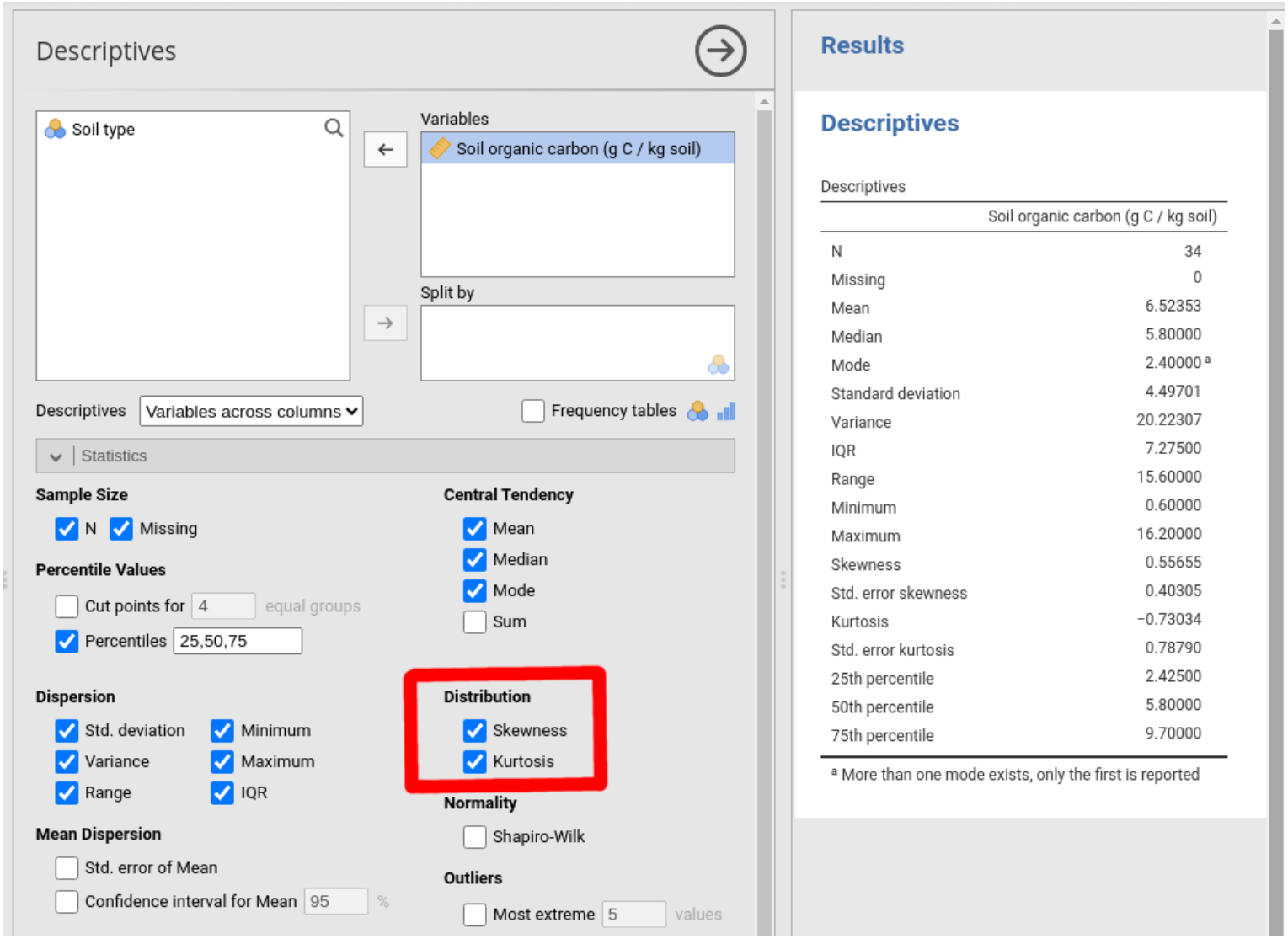
Often expressed as a percentage

x_1	x_2	x_3	x_4	x_5	x_6	x_7
17.1	15.2	14.9	12.6	15.2	10.3	12.7

$$CV = \frac{2.2598\,^{\circ}C}{14\,^{\circ}C} \times 100\% = 16.14\%$$

Useful for comparing variation across categories (e.g., species)

Descriptive statistics: Skew and kurtosis



Skew is the asymmetry of a distribution

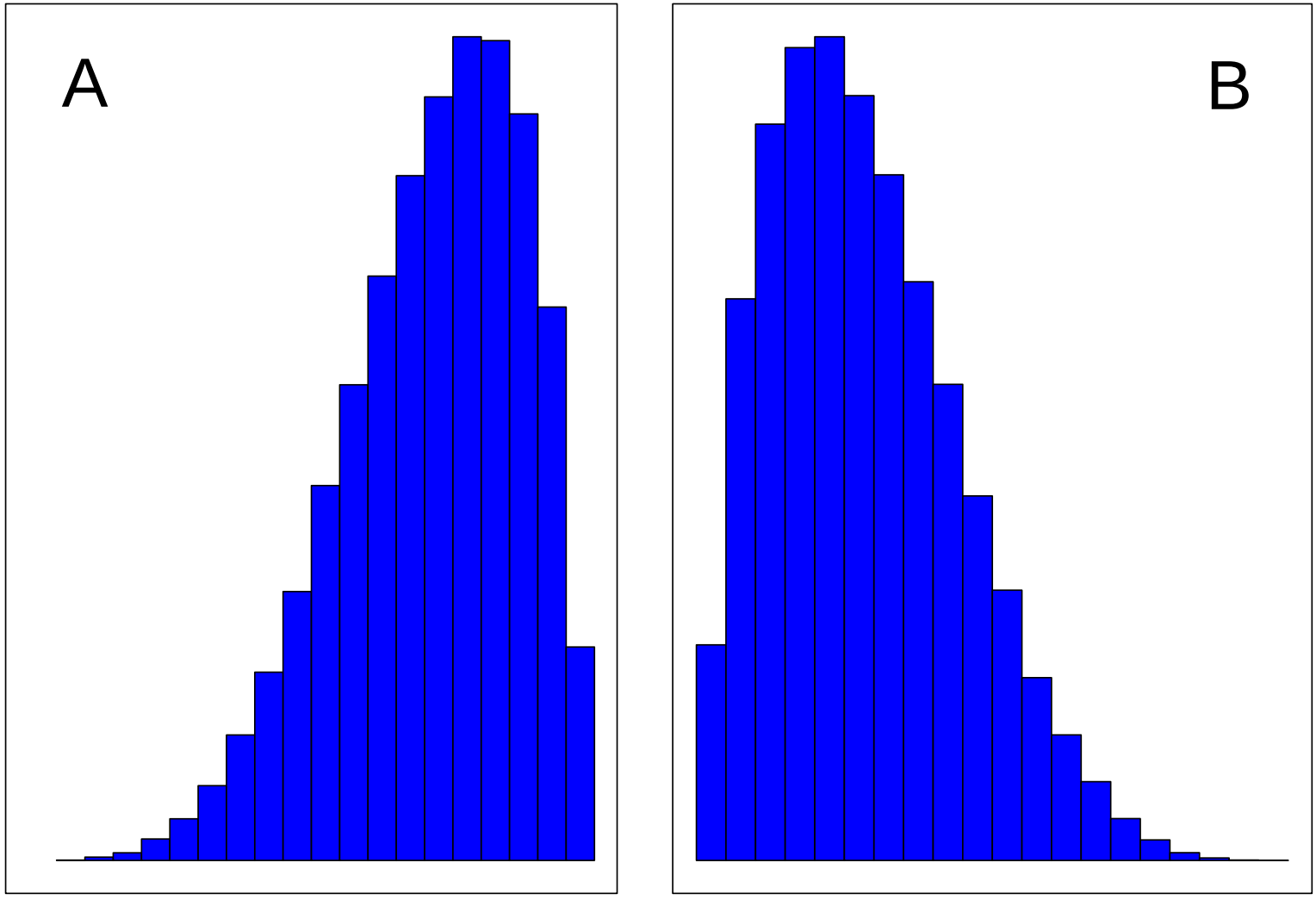


Figure 2: Histograms showing a (A) distribution that has a negative (i.e., 'left') skew and (B) distribution that has a positive (i.e., 'right') skew.

Kurtosis is the flattness of a distribution

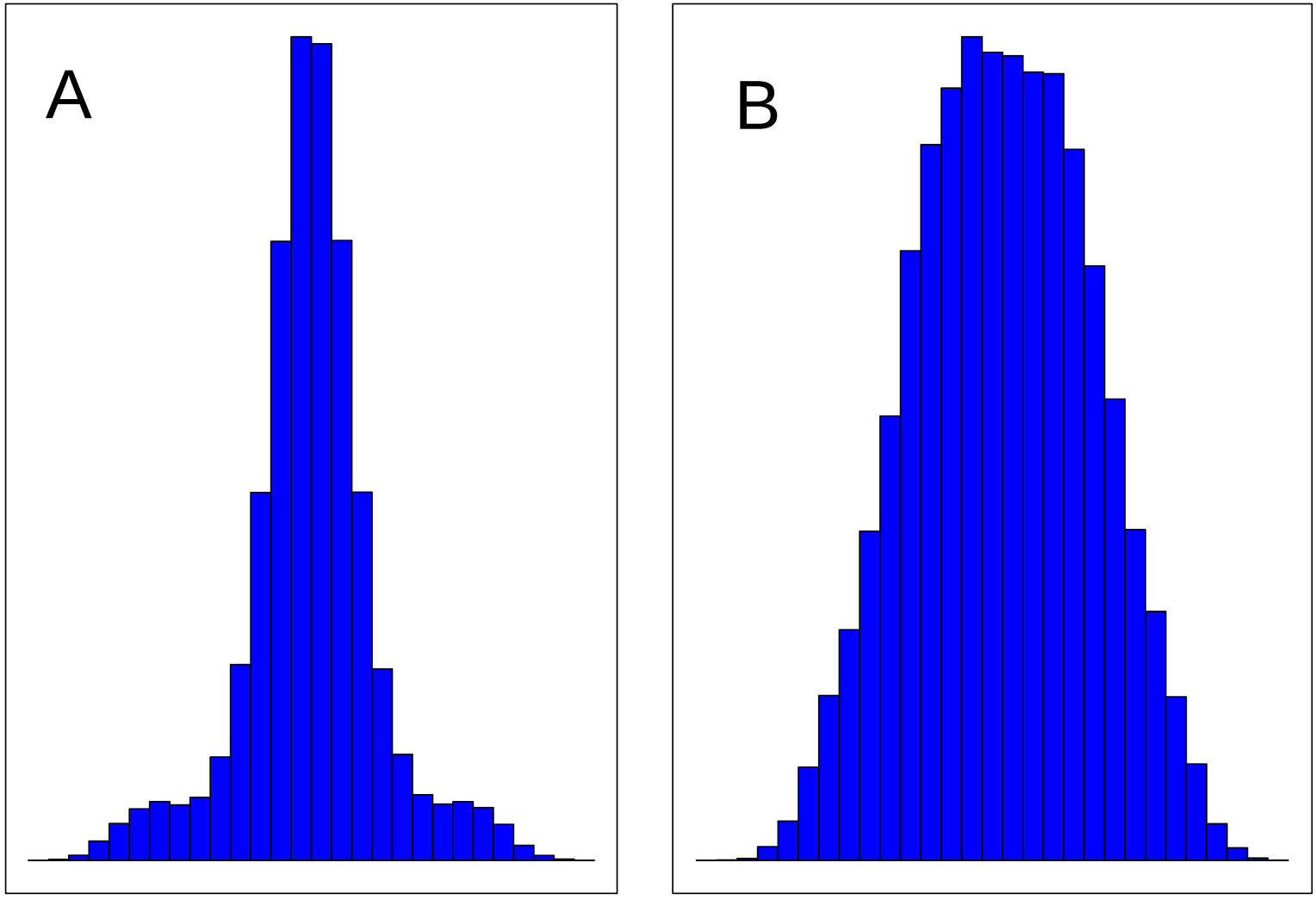


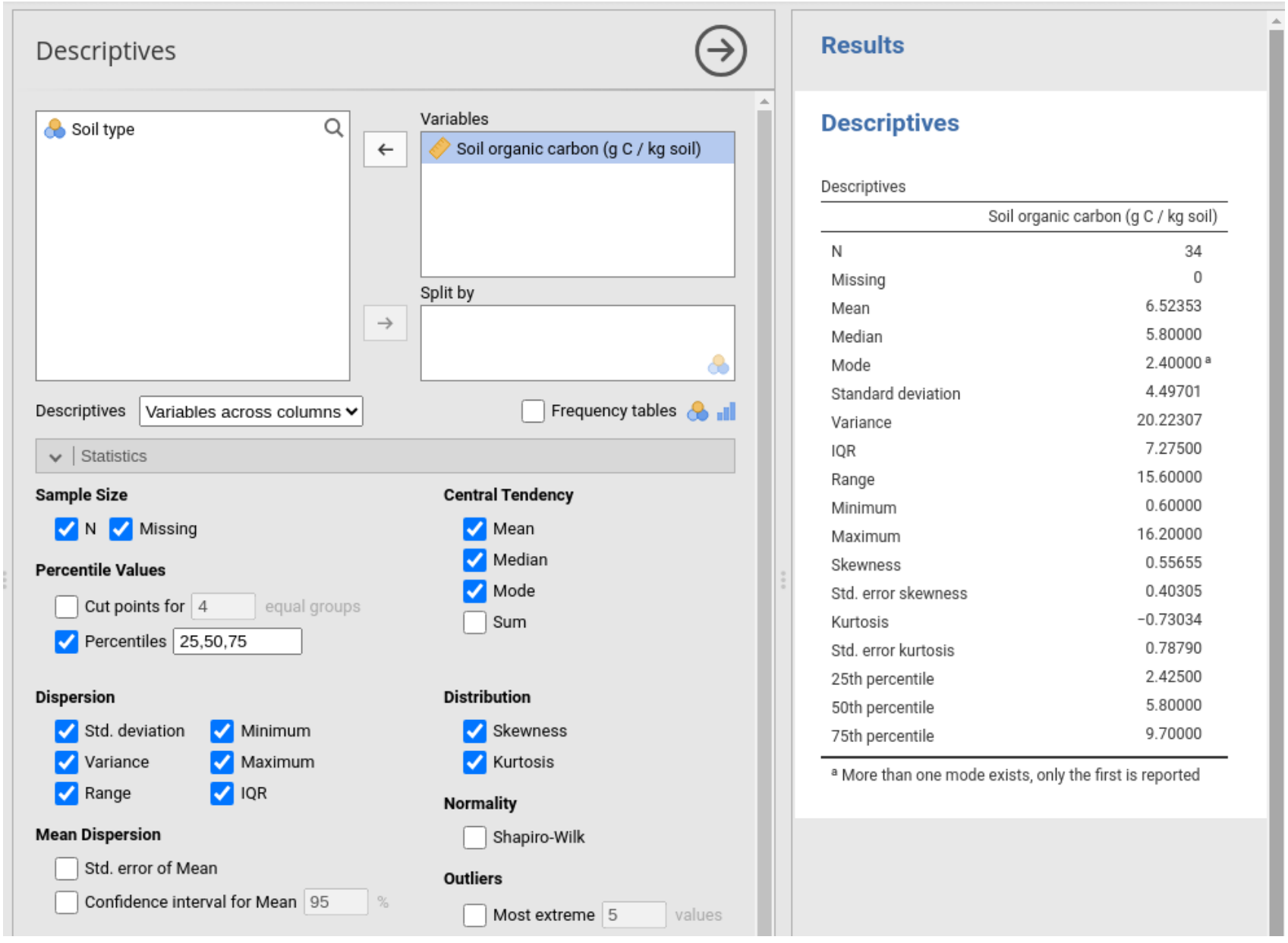
Figure 3: Histograms showing a (A) leptokurtic distribution and (B) platykurtic distribution.

Statistical moments

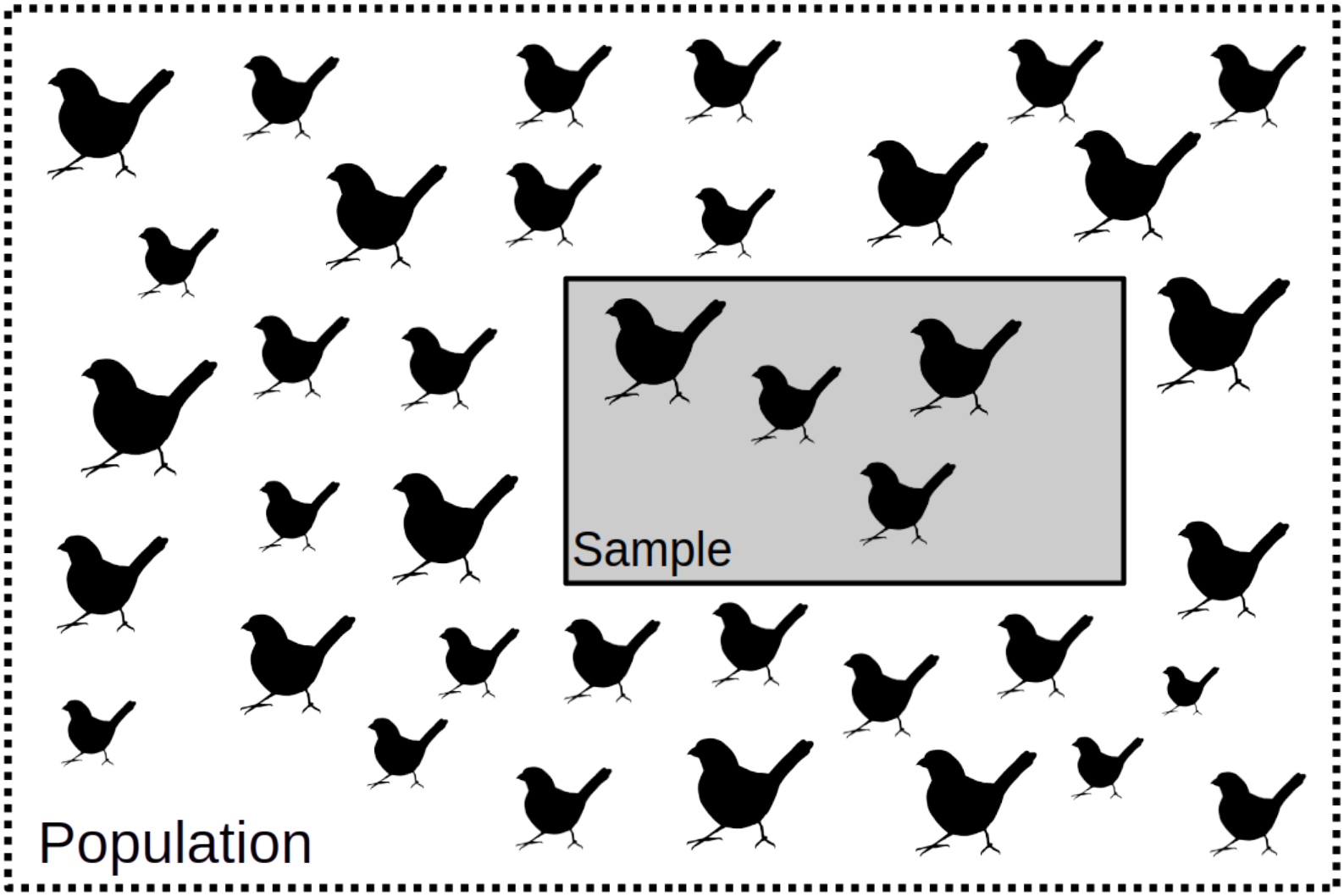
1. Mean
2. Variance
3. Skew
4. Kurtosis

Mathematically, deviations from mean raised to some power give the shape of a distribution.

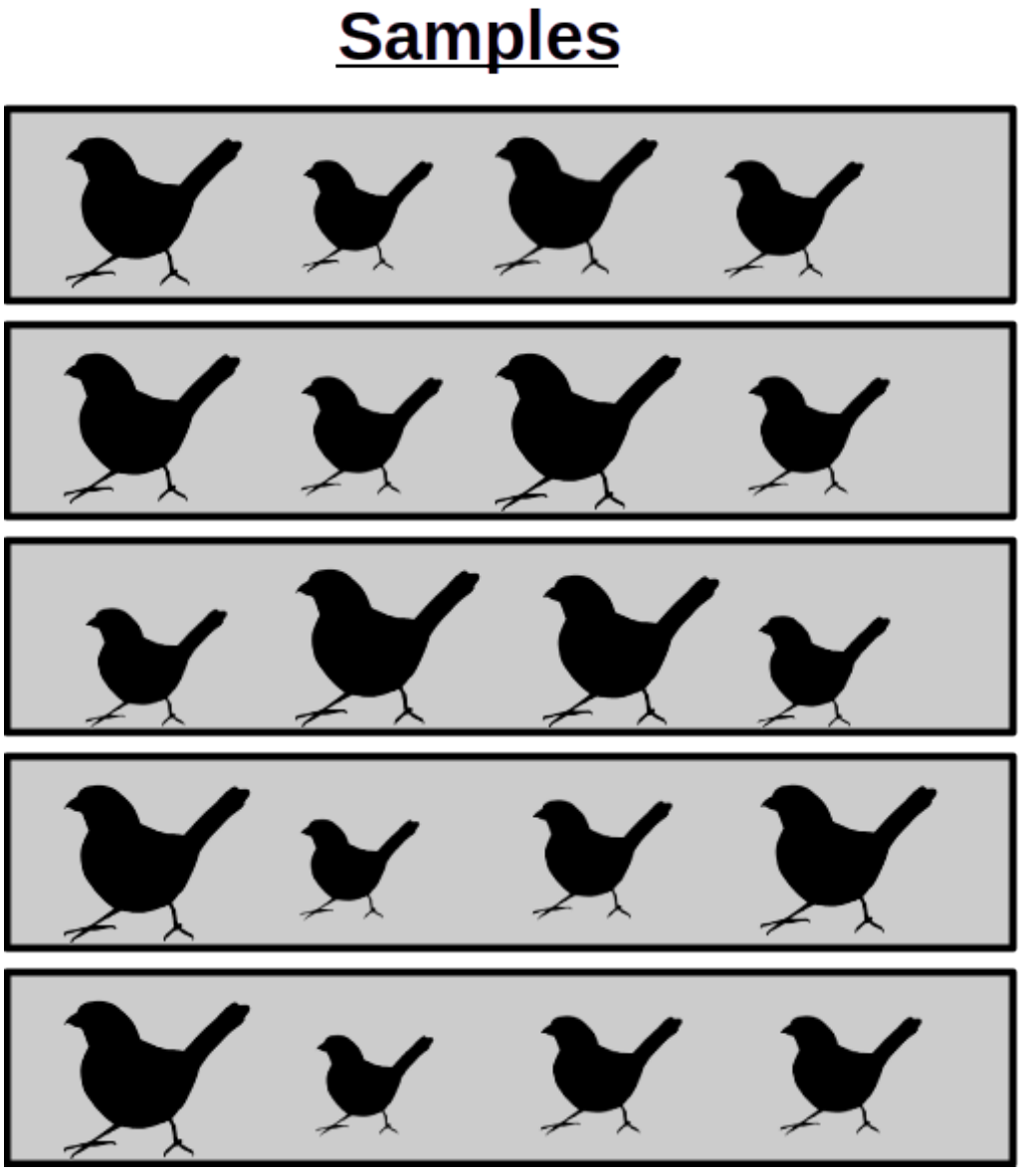
Descriptive statistics in jamovi



The standard error: Sample means and population mean



The standard error: Sample means and population mean

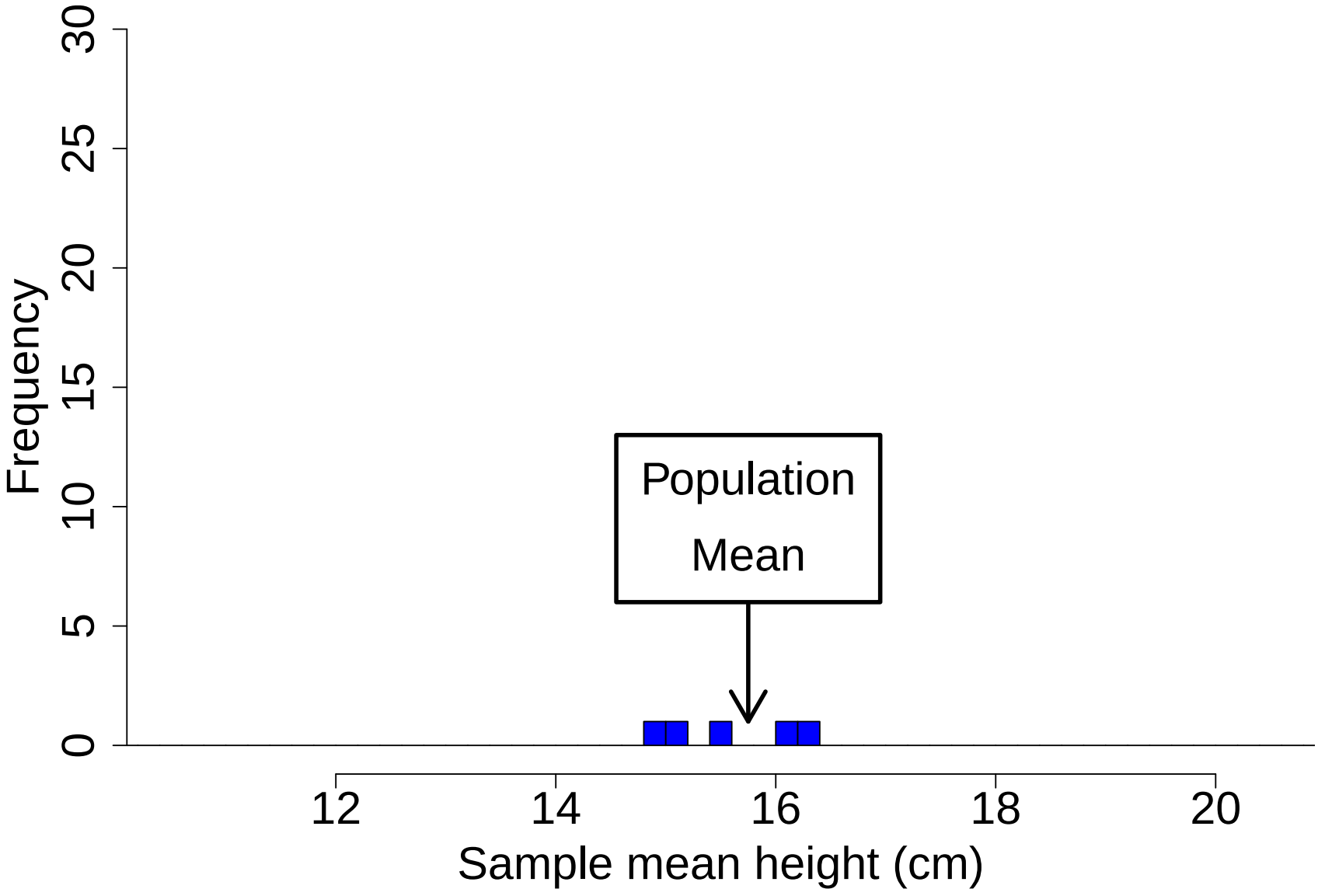


Repeated re-sampling

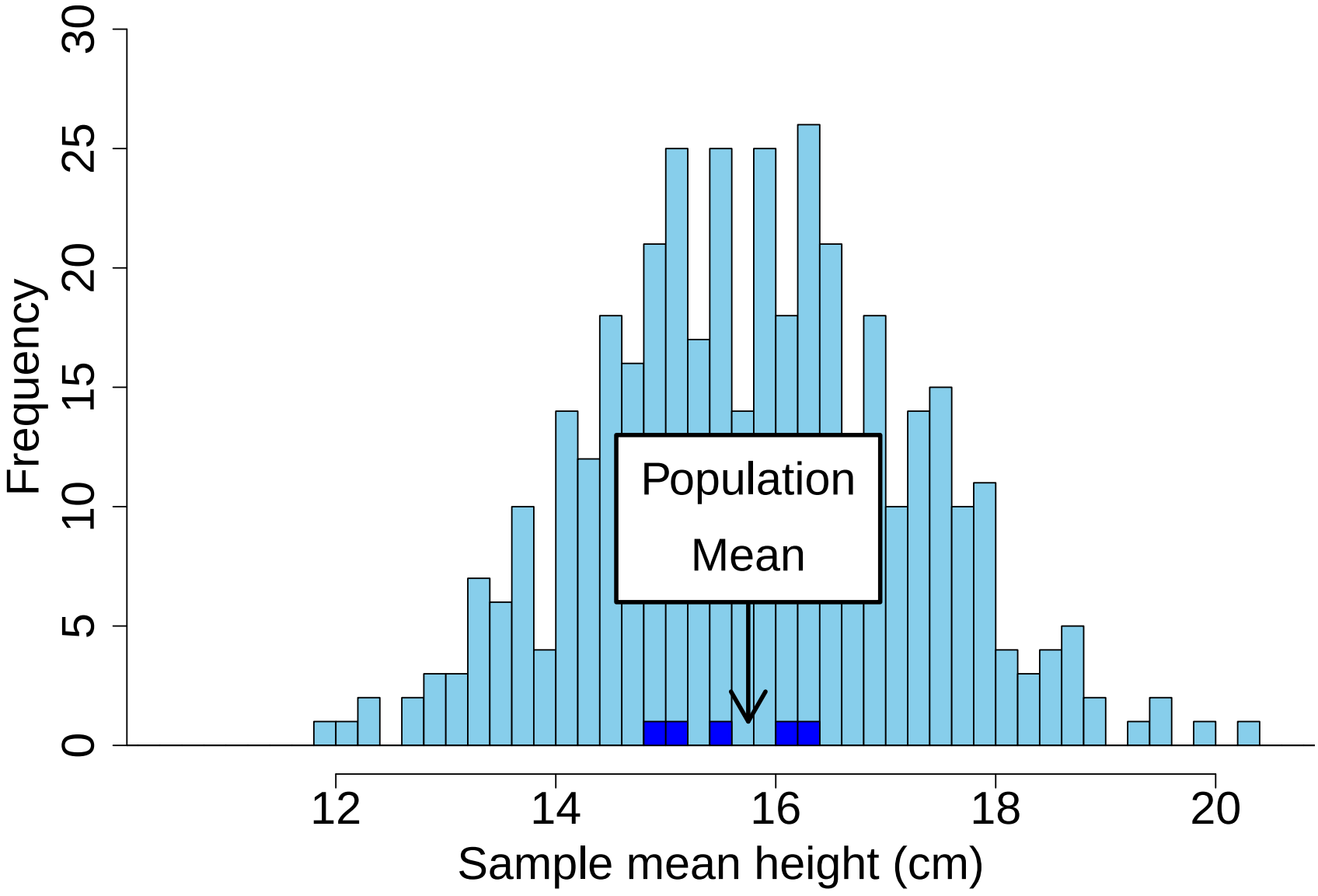
- What is our expectation?
- What is our uncertainty?

What is the **distribution** of the *sample mean* ($\bar{x}$) around the *population* mean (μ_x)?

The standard error: Sample means and population mean



The standard error: Sample means and population mean



The standard error: Sample means and population mean

- What is the **distribution** of the *sample mean* ($\bar{x}$) around the *population* mean (μ_x)?
- **Standard error**: The standard deviation of sample means around the population mean

The standard error: estimation

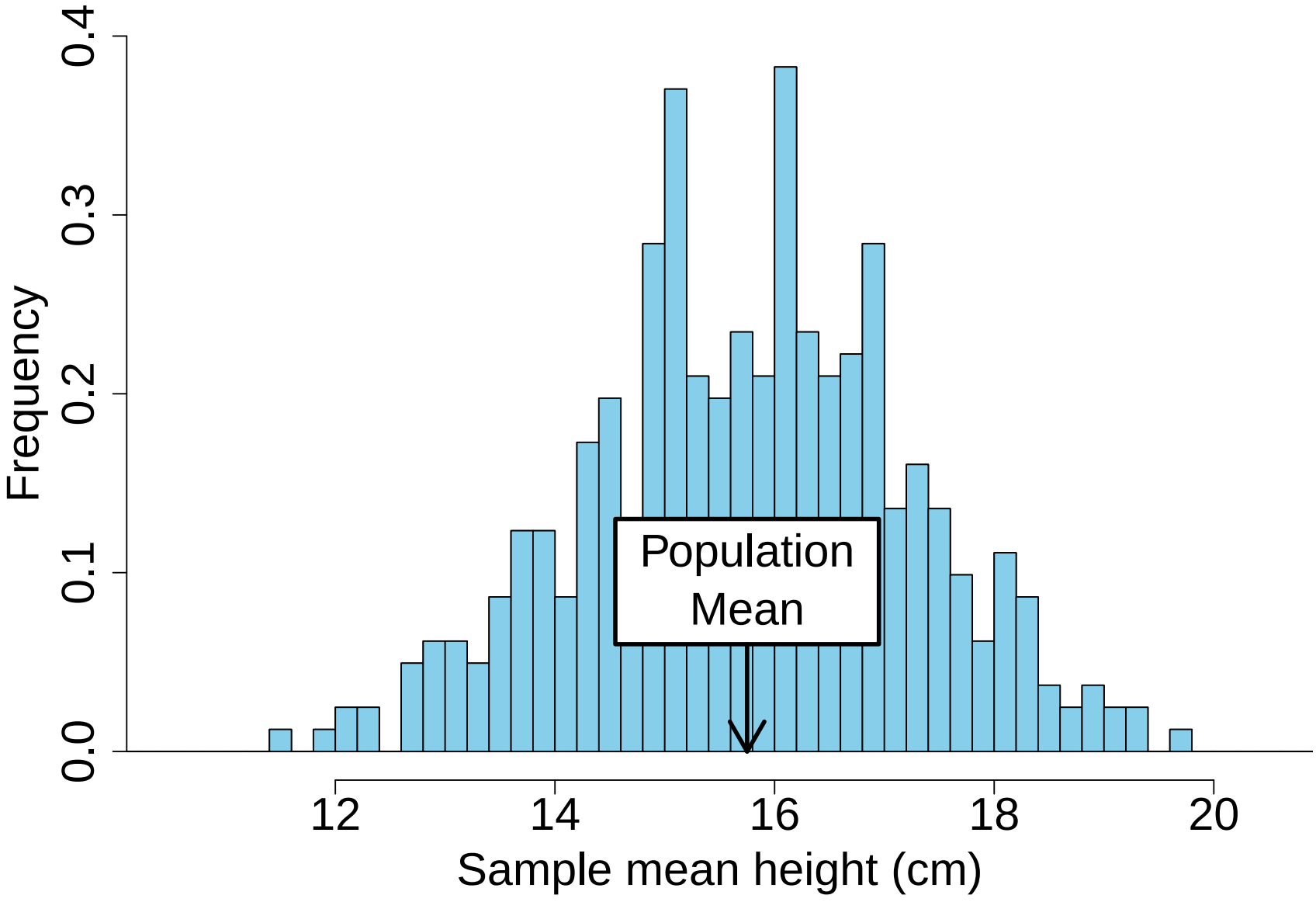
$$SE = \frac{Sample\ standard\ deviation}{\sqrt{Sample\ size}}$$

The standard error: estimation

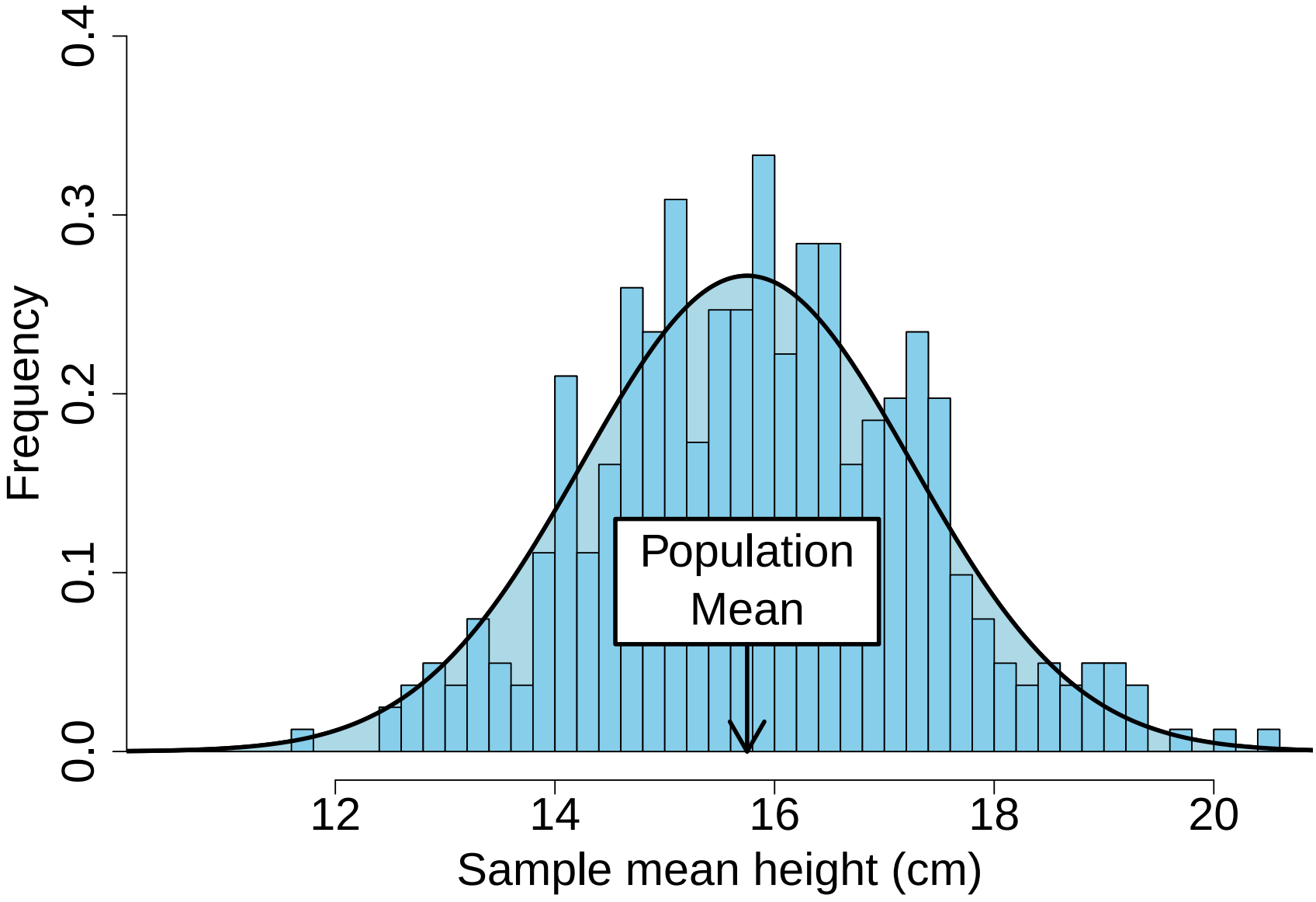
$$SE = \frac{Sample\ standard\ deviation}{\sqrt{Sample\ size}}$$

$$SE = \frac{s}{\sqrt{N}}$$

The standard error



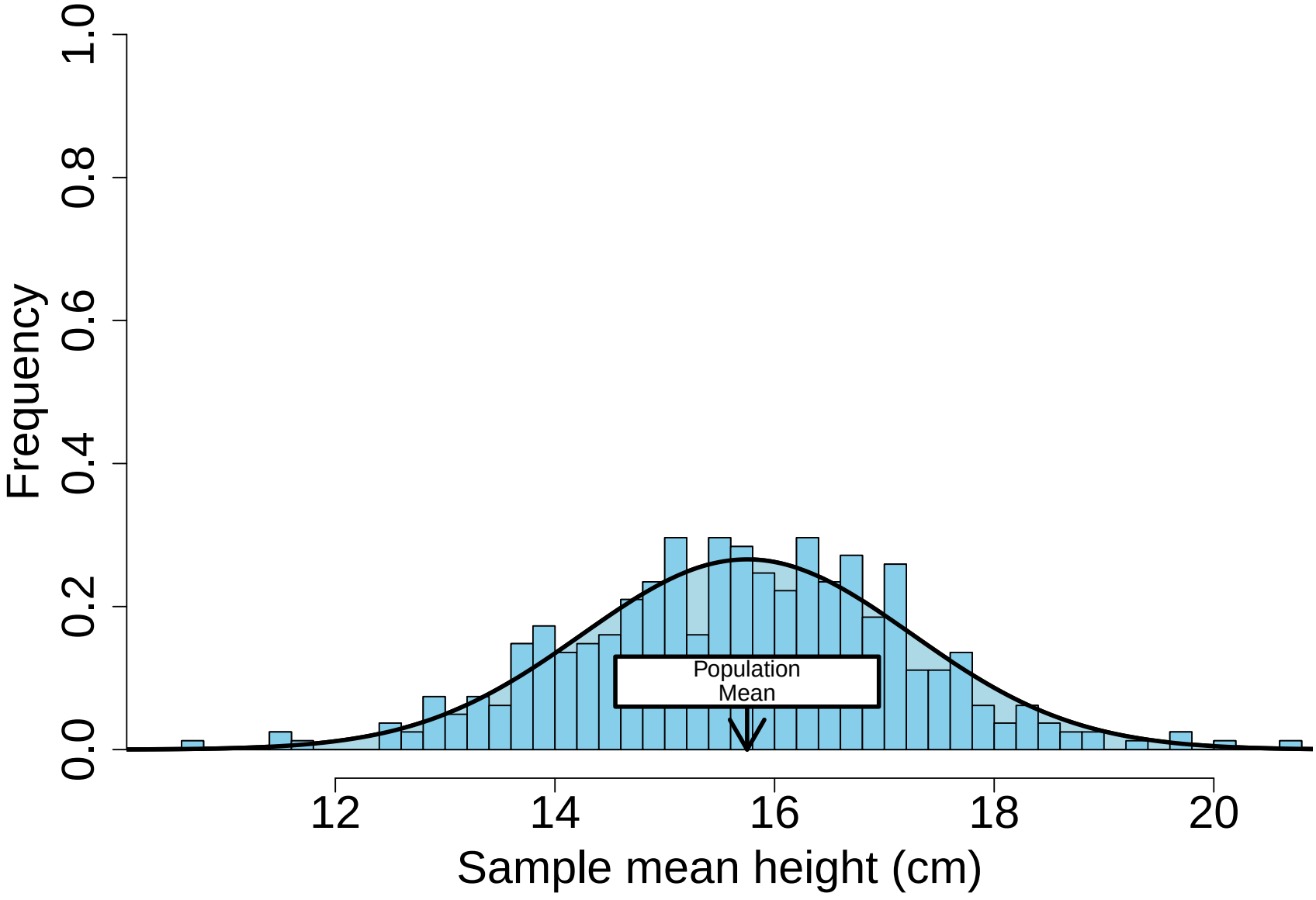
The standard error



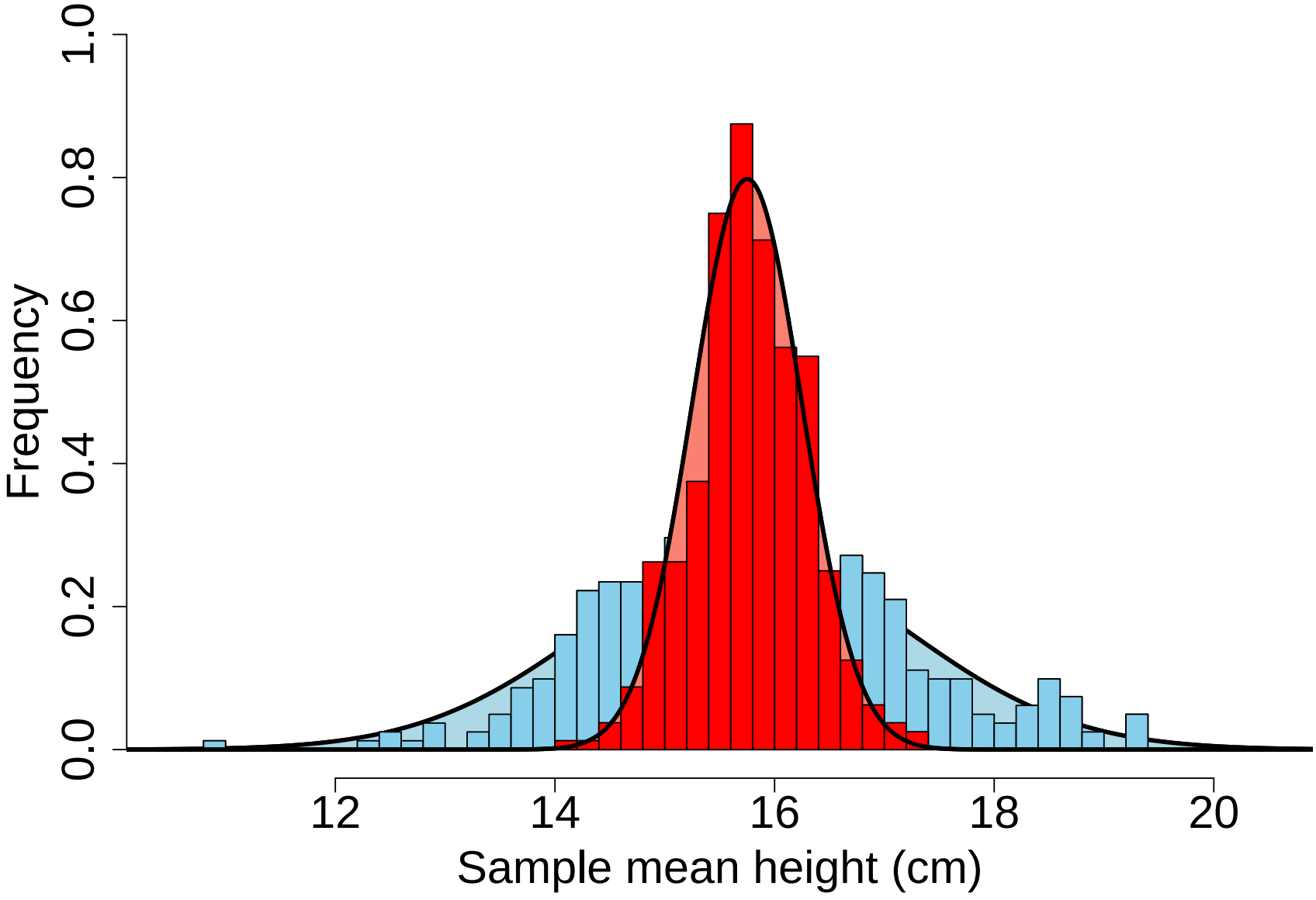
The standard error: Sample means and population mean

- **Standard error**: The standard deviation of sample means around the population mean
- Standard error measures the uncertainty of the sample mean

The standard error



The standard error



The standard error in jamovi