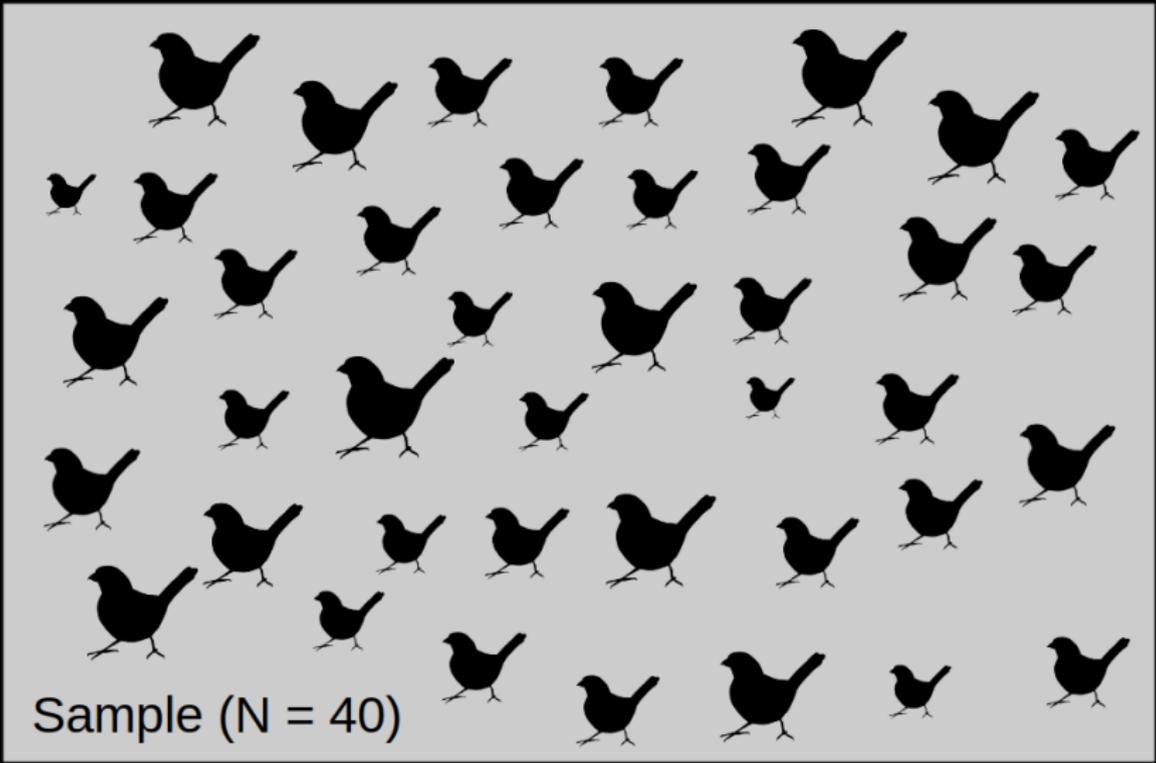


SCIU4T4: z- and t-intervals

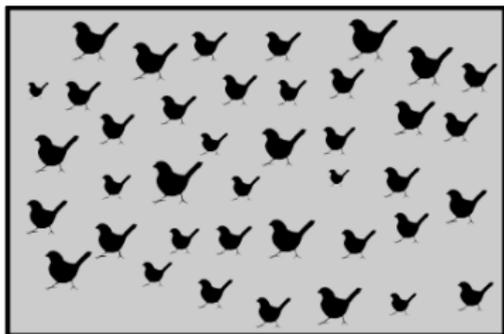
The goal of CIs is to contain uncertainty:

- ▶ How frequently will the CIs that we draw *contain* the population mean (μ_x)?
- ▶ Logic is again based on repeated resampling

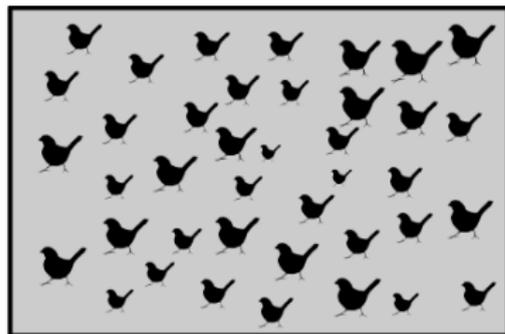
Consider the distribution of a sample of $N = 40$



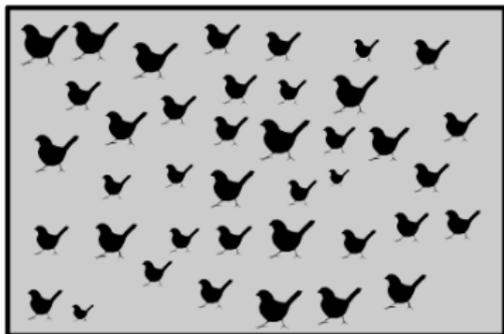
Sample means given true mean of 7.2 when $N = 40$



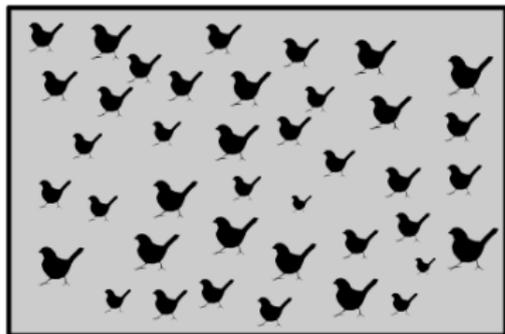
Sample mean = 6.59 cm



Sample mean = 7.45 cm

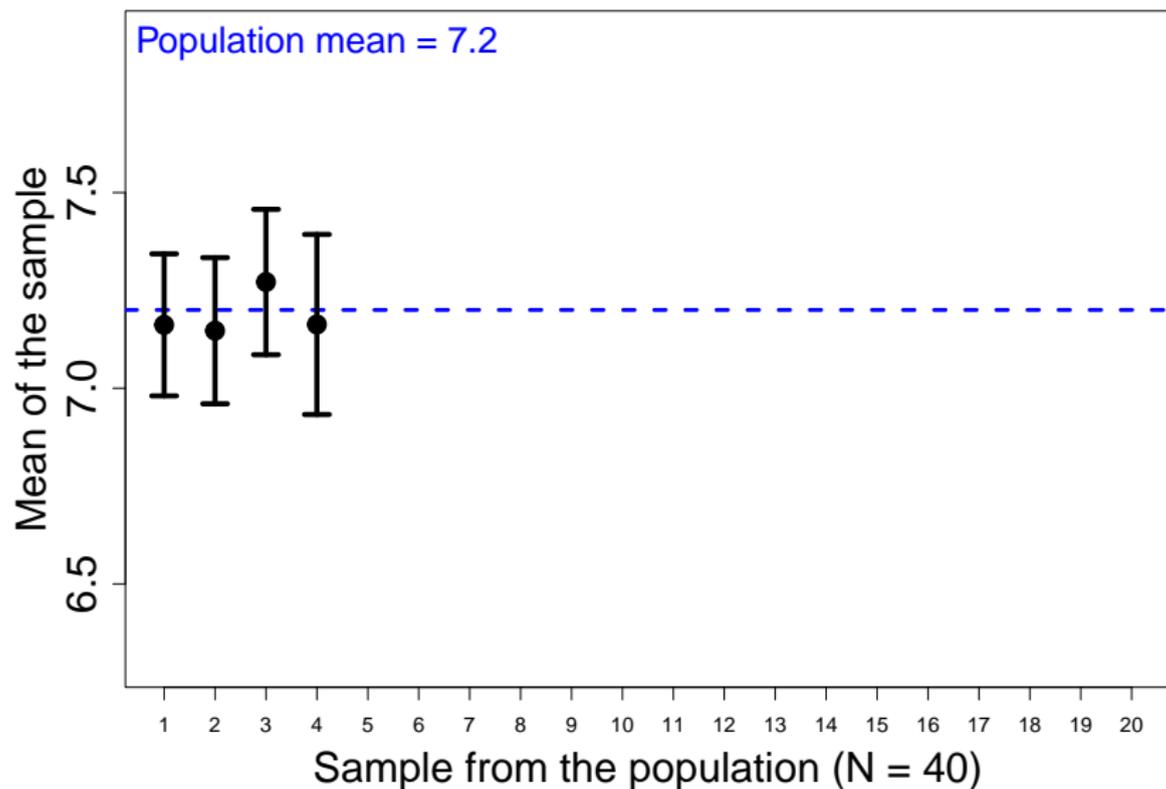


Sample mean = 7.03 cm

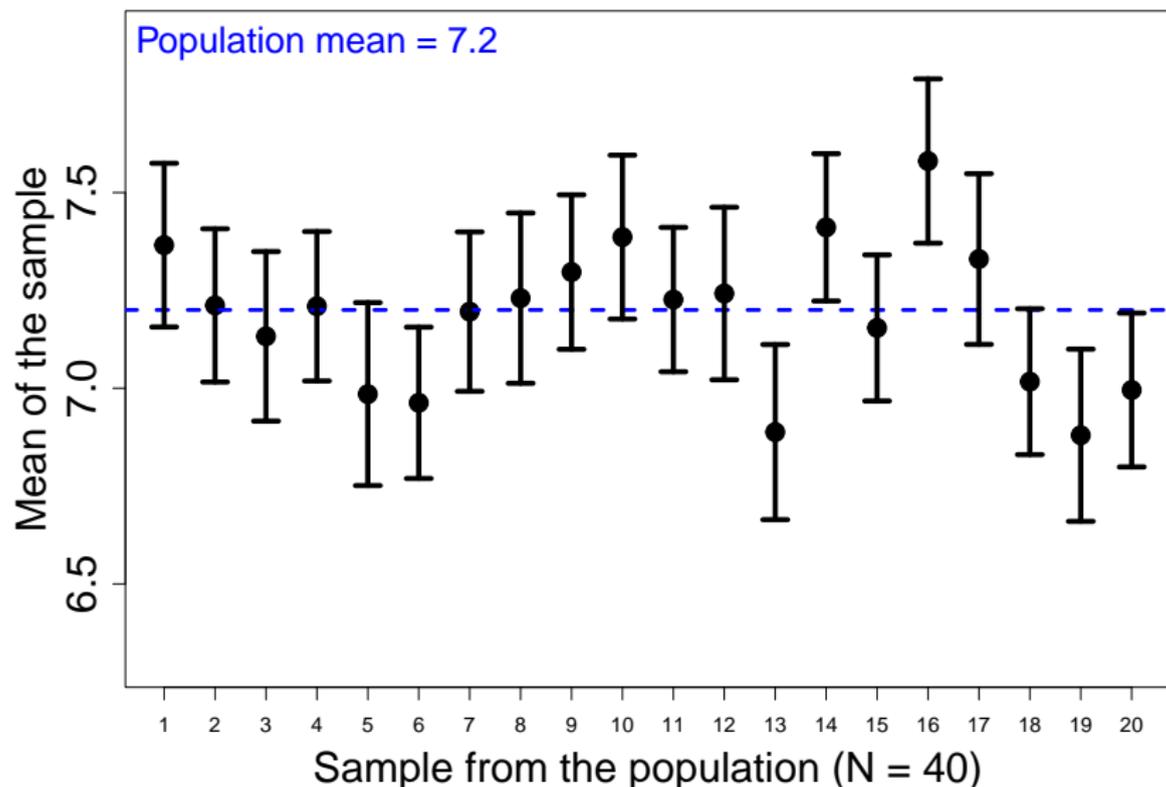


Sample mean = 6.50 cm

Confidence in a sampled mean value (80%)



Confidence in a sampled mean value (80%)



The goal of CIs is to contain uncertainty:

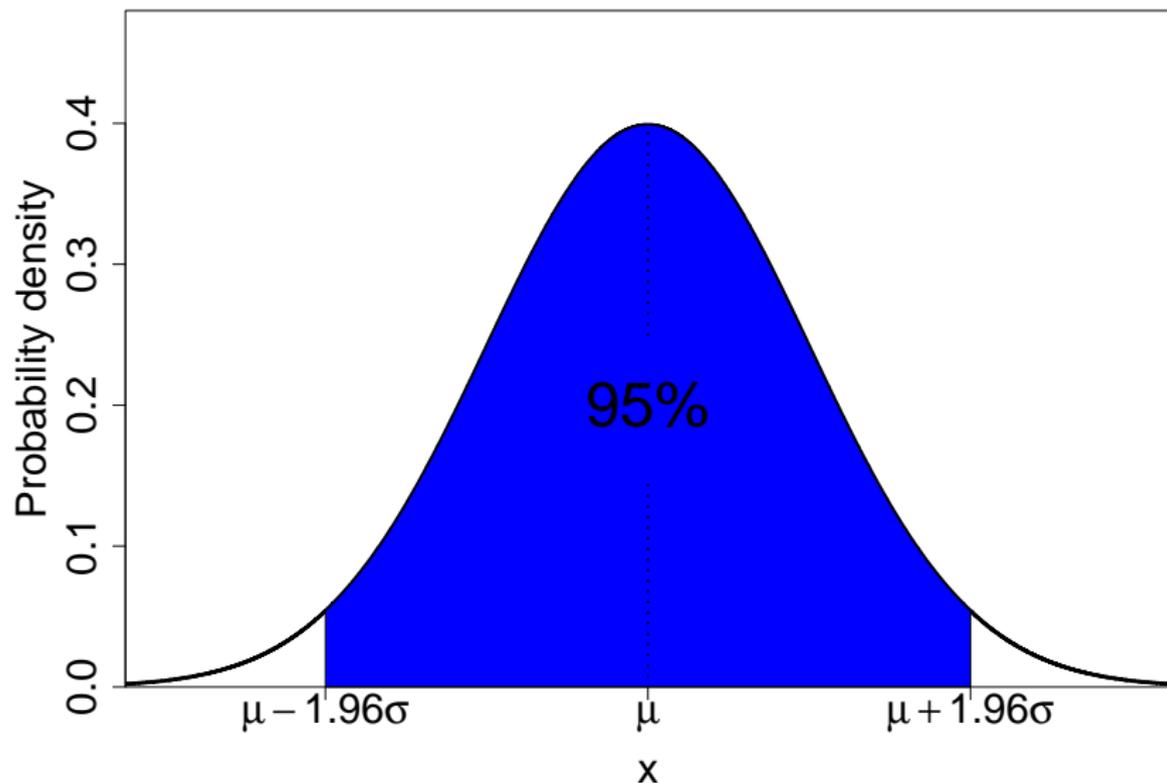
- ▶ How frequently will the CIs that we draw *contain* the population mean (μ_x)?
- ▶ Distribution probabilities (e.g., z-scores)
- ▶ **The standard error (s/\sqrt{N})**

Calculating CIs

▶
$$\text{LCI} = \bar{x} - z \left(\frac{s}{\sqrt{N}} \right)$$

▶
$$\text{UCI} = \bar{x} + z \left(\frac{s}{\sqrt{N}} \right)$$

Probabilities calculated from the normal distribution



¹<https://bradduthie.github.io/stats/app/zandp/>

Really assuming: $s = \sigma$

▶ $\text{LCI} = \bar{x} - z \left(\frac{\sigma}{\sqrt{N}} \right)$

▶ $\text{UCI} = \bar{x} + z \left(\frac{\sigma}{\sqrt{N}} \right)$

Problem assuming: $s = \sigma$

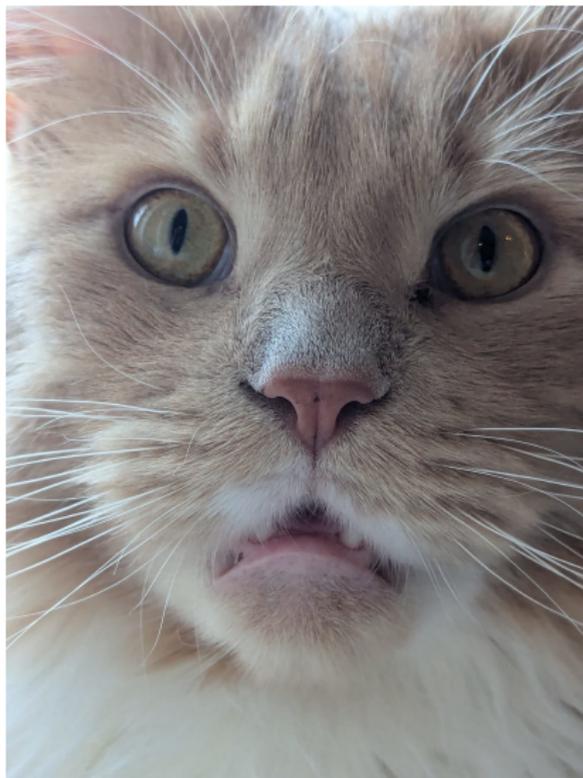
- ▶ Assuming no uncertainty in estimation of standard deviation (σ)
- ▶ If uncertainty in σ , our CIs will be wrong

Problem assuming: $s = \sigma$

- ▶ Problem less relevant¹ for large ($N > 30$) sample sizes
- ▶ Low sample size increases uncertainty of standard deviation

¹Sokal, RR & FJ Rohlf. 1995. Biometry. 3rd ed. WH Freeman & Company, New York, USA.

Confidence in a sampled mean value



Uncertainty in
the standard
deviation?

Need to adjust
the normal
distribution

The normal distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

¹You do not need to know this equation.

The t-distribution

$$f(t) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu}\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

¹Forget the equation, see: https://bradduthie.github.io/stats/app/t_score/

The t-distribution

$$f(t) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu}\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$



¹Forget the equation, see: https://bradduthie.github.io/stats/app/t_score/

Confidence intervals with a normal distribution

Verbally

- ▶ LCI = Mean - (z-score × standard error)
- ▶ UCI = Mean + (z-score × standard error)

Mathematically

- ▶ $LCI = \bar{x} - z \left(\frac{s}{\sqrt{N}} \right)$
- ▶ $UCI = \bar{x} + z \left(\frac{s}{\sqrt{N}} \right)$

¹<https://bradduthie.github.io/stats/app/zandp/>

Confidence intervals with a t-distribution

Verbally

- ▶ LCI = Mean - (t-score × standard error)
- ▶ UCI = Mean + (t-score × standard error)

Mathematically

- ▶ $LCI = \bar{x} - t \left(\frac{s}{\sqrt{N}} \right)$
- ▶ $UCI = \bar{x} + t \left(\frac{s}{\sqrt{N}} \right)$

¹https://bradduthie.github.io/stats/app/t_score/

Confidence intervals with normal distribution

x_1	x_2	x_3	x_4	x_5	x_6	x_7
17.1	15.2	14.9	12.6	15.2	10.3	12.7

$$N = 7 \quad \bar{x} = 14 \quad s = 2.26 \quad z = 1.96 \quad df = \infty$$

$$\begin{aligned} LCI &= \bar{x} - z \left(\frac{s}{\sqrt{N}} \right) \\ &= 14 - 1.96 \left(\frac{2.26}{\sqrt{7}} \right) \\ &= 12.34 \end{aligned}$$

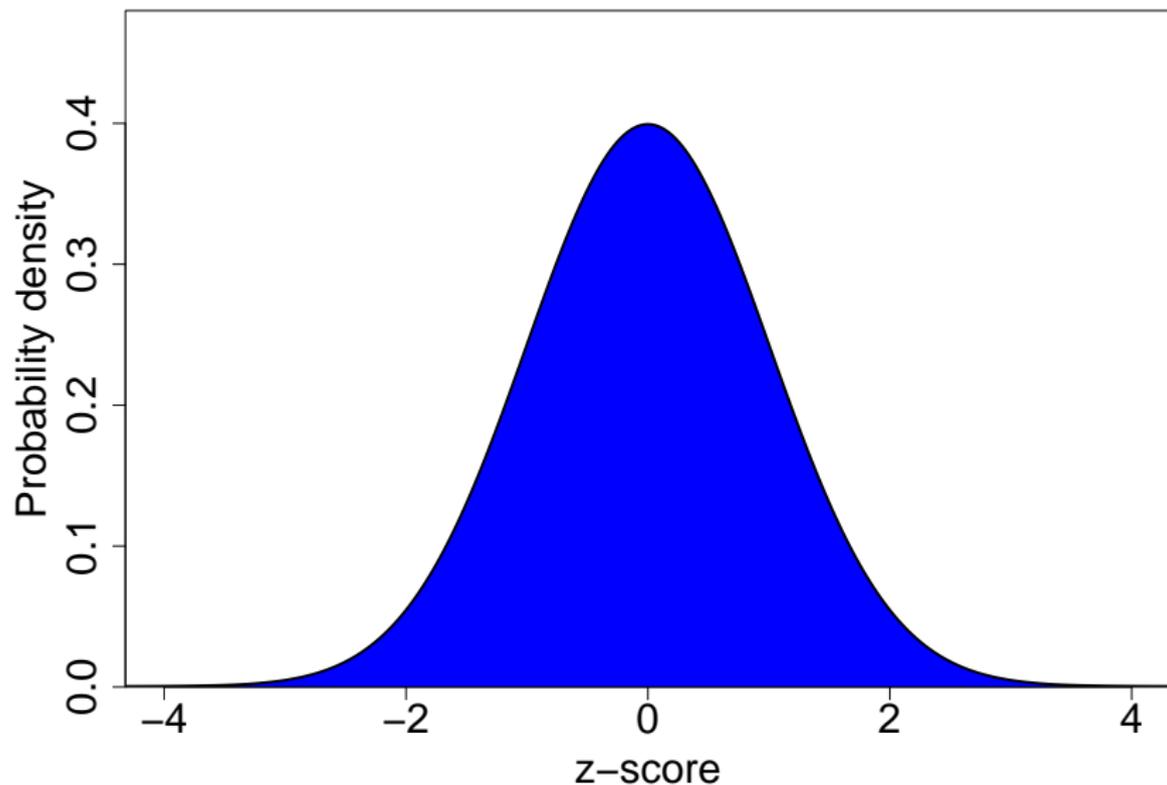
Confidence intervals with t-distribution

x_1	x_2	x_3	x_4	x_5	x_6	x_7
17.1	15.2	14.9	12.6	15.2	10.3	12.7

$$N = 7 \quad \bar{x} = 14 \quad s = 2.26 \quad t = 2.45 \quad df = 6$$

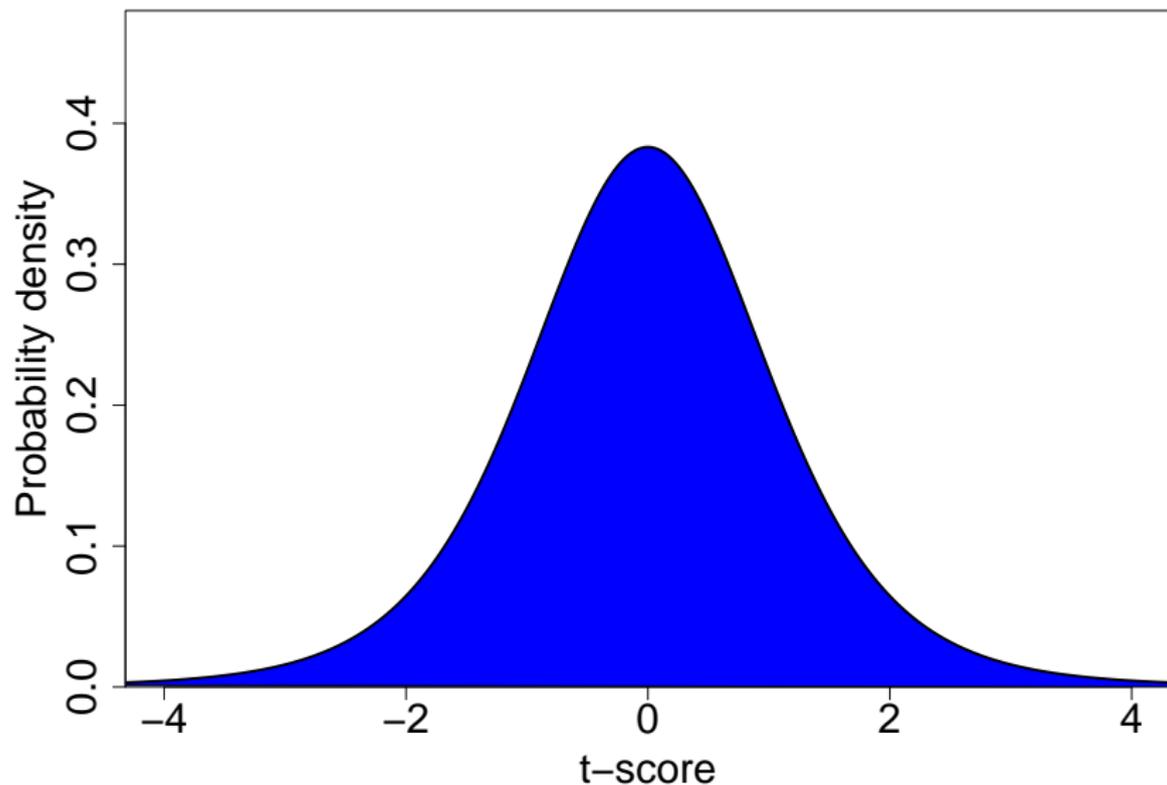
$$\begin{aligned} LCI &= \bar{x} - t \left(\frac{s}{\sqrt{N}} \right) \\ &= 14 - 2.45 \left(\frac{2.26}{\sqrt{7}} \right) \\ &= 11.91 \end{aligned}$$

Standard normal distribution



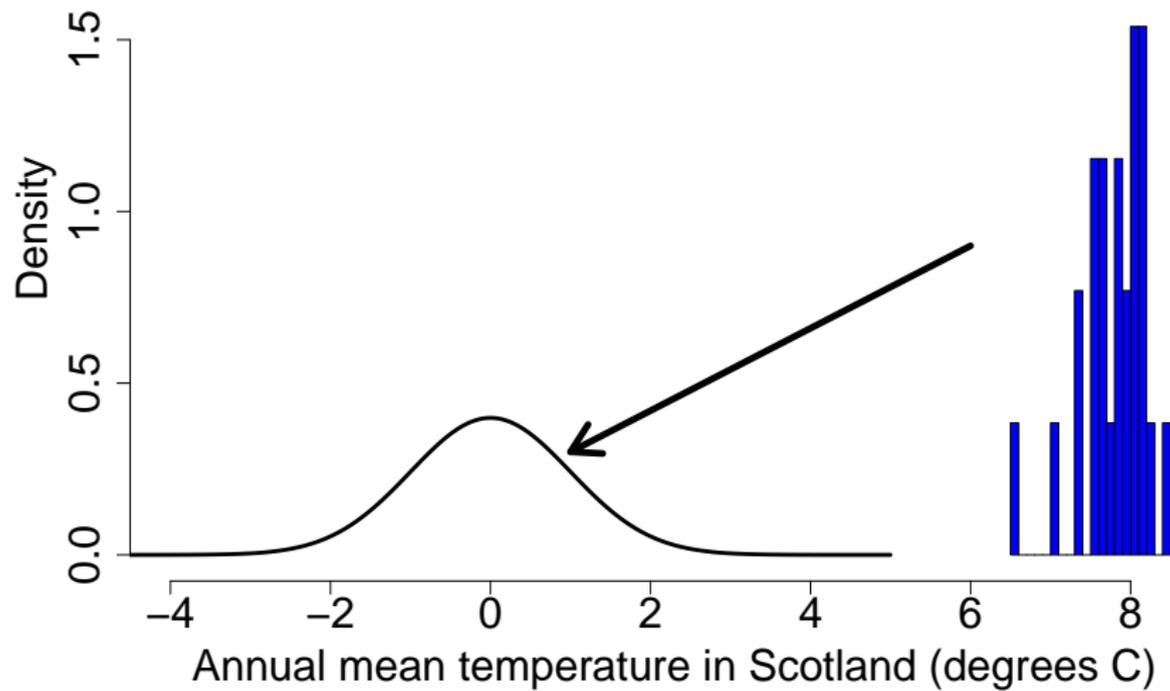
¹<https://bradduthie.github.io/stats/app/zandp/>

The t-distribution (6 df)



¹https://bradduthie.github.io/stats/app/t_score/

Turning a value into z-score



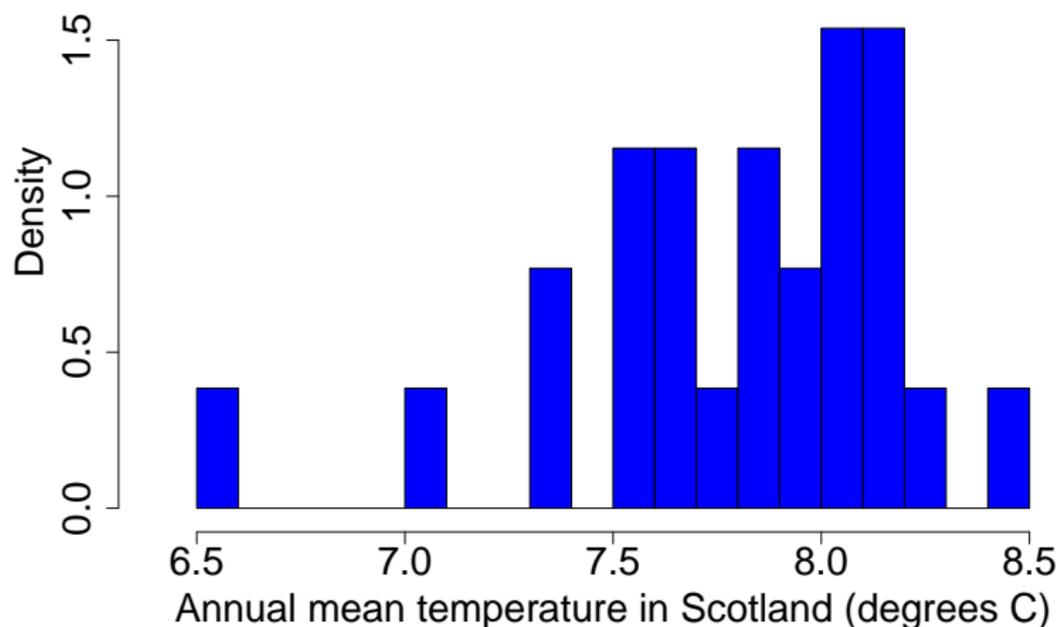
Translating a variable into a standard normal distribution

- ▶ Subtract mean
- ▶ Divide by standard deviation

$$z = \frac{x - \bar{x}}{s}$$

Can turn x-values into z-scores

What if the population mean is really 8.0?



$$N = 26 \quad \bar{x} = 7.789 \quad s = 0.4124844$$

¹Met Office. (July 1, 2021). Annual mean temperature in Scotland from 1995 to 2020. In Statista. Retrieved February 10, 2026, from <https://www.statista.com/statistics/367853/scotland-average-temperature/>

Testing a hypothesis with the t-distribution

- ▶ Subtract hypothesised mean
- ▶ Divide by standard deviation

$$t = \frac{\bar{x} - \mu}{SE(x)}$$

If μ is the population mean, what is the probability of getting \bar{x} ?

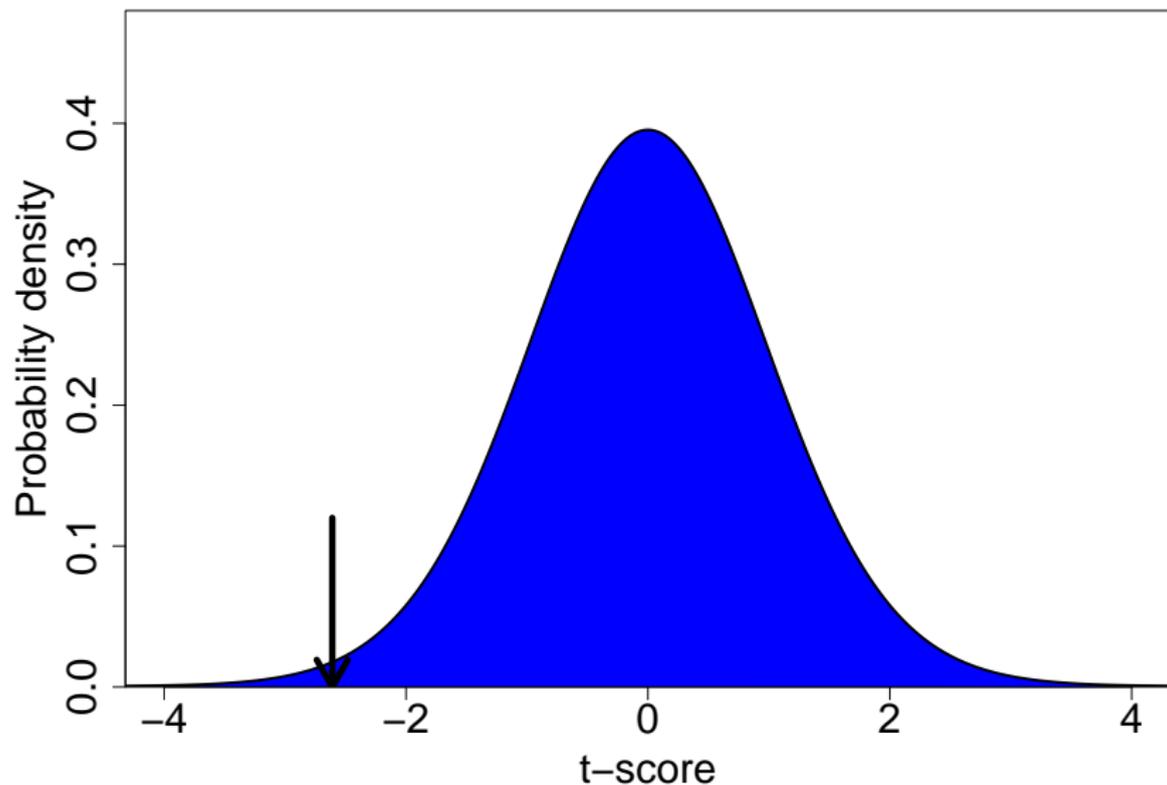
Testing a hypothesis with the t-distribution

- ▶ Subtract hypothesised mean
- ▶ Divide by standard deviation

$$t = \frac{7.789 - 8}{(0.4124844 / \sqrt{26})} = -2.61$$

If 8 is the population mean, what is the probability of getting 7.789?

The t-distribution (25 df)



¹https://bradduthie.github.io/stats/app/t_score/